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THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

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From a snap-shot by Dorothy Wallace in 1908

THE
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VOLUME LIV — JULY 1921 — NUMBER 1

SHERBURNE WESLEY BURNHAM, 1838–1921

By EDWIN B. FROST

Sherburne Wesley Burnham, senior astronomer at the Yerkes Observatory from its opening in 1897 until his retirement in 1914, died at his home in Chicago on March 11, 1921, at the age of eighty-two years.

He leaves an imperishable memory in the history of stellar astronomy, for the great number of new double stars which he discovered, for the vast quantity of precise measurements of such objects which he made, and for his monumental work of critical compilation. His symbol β is recognized throughout the astronomical world as the synonym of scientific precision and of remarkable visual discrimination.

Mr. Burnham was a "self-made man." This term is somewhat overworked, perhaps particularly on the Continent of Europe, where it is supposed to be rather typical of successful Americans. It expresses the fact that he did not have the advantage of any studies at a college or university in mathematics or astronomy, nor did he have, during the years when he was acquiring his knowledge in this department, much direct contact with professional astronomers, from whom he could obtain advice. He had to go to such books as he could find in the libraries in Chicago and by his own

efforts acquaint himself with the methods of observing and the literature of astronomy in the field which he pursued.

After some study at the academy in the town of his birth, Thetford, Vermont, in the hill country of New England, he practiced shorthand by himself, and in 1858 went to New York, where he found employment in business. Later, during the Civil War, he was in New Orleans with the Union troops, serving as an official reporter for the military government, and subsequently he was similarly employed at several Constitutional Conventions in the southern states. After the war he came to Chicago and continued as official court reporter for more than twenty years. This labor of recording the verbal testimony before the United States courts and then writing it out in longhand was indeed arduous, and involved no small strain upon his eyes; but for a score of years he observed at night as assiduously as if he had no engagements for the day.

His interest in astronomy had been awakened by the possession of two small telescopes in succession, and he ordered from the Clarks in 1869 a 6-inch refractor, with the stipulation that it should be as good as they could make. After this telescope was delivered in 1870, his attention was directed to double stars and it appeared that he had a most extraordinary keenness of vision, particularly in this discrimination as to the character of a stellar image. The objective was a fine one, perhaps a prototype for the subsequent masterpieces of optical skill made by the Clarks.

In 1873 and 1874, the Royal Astronomical Society published in the *Monthly Notices* five lists of new double stars by Mr. Burnham. He had obtained great assistance from Webb's *Celestial Objects for Common Telescopes* and had entered into correspondence with Mr. Webb, whose encouragement was of much value to him and led to his connection with the Royal Astronomical Society, of which he became a member in 1874. With the 6-inch telescope he discovered 451 new double stars, many of which were so close as to give difficulty to other observers having much larger instruments. The forty-fourth volume of the *Memoirs* of the Royal Astronomical Society, published in 1879, contained the measures of 250 new doubles, with micrometric observations of 250 other pairs. For about six years of the period between 1877 and 1884,

Mr. Burnham had the opportunity to use the fine $18\frac{1}{2}$-inch refractor of the Dearborn Observatory, then located in the city at the former Chicago University; and for part of the time he held, at least nominally, the office of director of that observatory. During part of the year 1881 he was associated with the then new Washburn Observatory of the University of Wisconsin at Madison, of which Professor E. S. Holden was the director. There he observed both with the $15\frac{1}{2}$-inch telescope and his 6-inch, the latter being purchased by the University of Wisconsin as a part of its equipment.

When the Lick Observatory was opened in 1888, the position of astronomer was offered to him and was promptly accepted, although no doubt at a very considerable pecuniary sacrifice, especially for a man with a family of six children. The Lick Observatory was not unfamiliar to him, as he had been there twice before, on commissions from the Lick Trustees: once for two months in 1879, to test (with his 6-inch refractor) the conditions of observing on Mount Hamilton, which had been chosen as the site for the new observatory; and in 1881, to observe the Transit of Mercury, for which he was able to use the 12-inch telescope, then already mounted. He carried out a fruitful program of observations of double stars at the Lick Observatory, with the use of the 36-inch and 12-inch refractors for four years. This was only interrupted by an expedition to Cayenne, to observe the eclipse of December 21, 1889. Internal conditions had developed at the observatory which were not agreeable to Mr. Burnham and he resigned his position in June, 1892, and returned to Chicago, where he was offered the office of clerk of the United States Circuit Court. His duties in this position were highly responsible, but perhaps less exacting than they had been previously when he was court reporter. He discharged these duties with great accuracy and ability, and his services were highly appreciated by the judges and other officers of the federal court, and by a wide circle of men in the legal profession. He, in turn, had great respect, amounting in some cases to veneration, for many of the distinguished jurists presiding in the courts in which he served during his terms of office in both capacities. From 1897 to 1902, he also acted as receiver for the Chicago & Northern Pacific Railroad.

For a period of five years, beginning in 1892, Mr. Burnham was not directly connected with an observatory and his use of a telescope was limited to occasional visits to the Dearborn Observatory, then at Evanston under the directorship of Hough. Burnham's interest in double stars was not in the least diminished, however, and he published a dozen papers in the *Monthly Notices*, chiefly on the orbits of stars which he had previously observed. He was also occupied with his manuscript catalogue of all double stars, which he had begun many years before.

In 1892 the University of Chicago received from Mr. Yerkes the promise that he would present to it a well-equipped observatory, and the disks for the 40-inch refractor were purchased by him. There were delays in completing the plans for the observatory, however, so that the building could not be begun until 1895; but meanwhile Mr. Burnham was appointed astronomer, with the title of Professor of Practical Astronomy, in the University of Chicago. His actual work was not begun, however, until after the formal opening of the observatory, in October, 1897, when he was assigned two nights a week with the great telescope. He commonly came to the observatory on the afternoon of Saturday and observed through that night and through Sunday night, taking what rest he could in his office during the intervening day; but he was off for his duties at the court on the early train on Monday morning, carrying with him his records of observations, which were kept in a most systematic manner and were promptly reduced. His observing program was selected with great care and his methods were such that he worked very efficiently and rapidly. He greatly preferred to observe toward the east and rarely reversed the telescope to follow a star which had gotten away from him to the west. Under exceptional circumstances he measured as many as 100 pairs in a single night with the forty-inch telescope! He seldom allowed himself to be distracted by other astronomical objects, however interesting, but he occasionally measured nebulae which came within the field. He made no search for new double stars after he began work with the 40-inch telescope, rightly judging that it was more important to follow up the difficult pairs which were beyond the reach of many telescopes, but which were of the greatest interest because of the rapid motion often revealed. He soon collected his

own measures and those of some other observers of 1290 double stars which he had discovered between 1871 and 1899, and these were published in 1900, as Volume I of the *Publications of the Yerkes Observatory*.

In 1902 he resigned his position as clerk of the court, despite the fact that he could have availed himself of its life-tenure. This gave him all of his time for astronomical work, which had never been the case before except for the four years when he was at the Lick Observatory and for the six months at the Washburn Observatory. He did not move out to Williams Bay, however, preferring to maintain the family home in Chicago, where it had been so long established.

With this additional freedom he thus had further opportunity for completing his great *General Catalogue of Double Stars within 121° of the North Pole*, which was now in its third edition as a manuscript. Attempts had been made for some years to find means for publishing this, but it was not until 1906 that this could be done. The recently endowed Carnegie Institution of Washington undertook in 1905 the publication of the work, and the printing was begun at the University of Chicago Press, where Mr. Burnham personally supervised the composition. The preparation of this comprehensive work constantly brought to light objects requiring new observations before their inclusion in the catalogue; thus most of his observations at the 40-inch telescope, from 1900 to 1906, nearly 10,000 in number, found their original publication in this catalogue. The first of these two stately quarto volumes summarizes the statistical information as to 13,665 double stars, in 275 pages; the second gives in 1086 pages the detailed information as to all the important observations of these pairs, with diagrams of the orbits of many of them. Aside from its large amount of original data this is not a mere compilation, for Professor Burnham's experienced discrimination is seen throughout the work; while he gave references to the complete literature for every pair, he published in detail only the data of observers whose work was of a high standard of excellence.

Although Mr. Burnham never did any actual teaching and, so far as we know, never gave a public lecture, nevertheless by his fine example of persistent devotion to his special line of research, and

by his untiring work of observation, even at an age when most men would have availed themselves of the privilege of retiring, he led the way in America in the field of double stars and has been followed by a worthy band, who gladly recognized him as master. For many years Mr. Burnham gave freely his valuable counsel to many younger observers in making up their programs for observations of double stars.

After the publication of the great catalogue, Mr. Burnham directed his observations largely to the micrometric reference of stars, chiefly from the *General Catalogue*, to faint stars in their vicinity, with a view to determining the relative proper motion, because he fully appreciated the great superiority of such observations with a telescope having a focal length of 19 meters over those made with meridian circles. His results, which included nearly 10,000 measures, were published in 1913 in a quarto volume of 311 pages by the Carnegie Institution of Washington, with the title *Measures of Proper Motion Stars Made with the 40-Inch Refractor of the Yerkes Observatory in the Years 1907–1912*. These measures lay the foundation for a very much better knowledge of relative proper motions of these stars as they may be remeasured in future years. Such observations of stars did not require the high standard of "seeing" necessary for close double stars with a big instrument, and there is also a possibility that there was some slight falling off in the keenness of β's vision after he was seventy years of age.

Physically Mr. Burnham was slight and of moderate stature, but all his muscles were at his command. The co-ordination of his hand and eye had always been notable. Thus he was an expert shot with the rifle, unusually good at bowling, and generally a man who mastered whatever he undertook. His endurance was exceptional, whether it was shown in climbing about the cañons near the Lick Observatory, or in long rides on his bicycle, or in carrying on his work at the great telescope during thirteen or even fourteen hours of a cold winter's night. He was abstemious in his habits and took very little food during his visits to the Observatory, but he had constant comfort from his cigar. The photograph which accompanies this sketch is taken from a snap-shot made while he

was at his ease near the observatory, when he was about seventy years of age.

He was an expert with the camera and had won the prizes at exhibits in the earlier days, but there are very few portraits of himself available.

He knew nothing of bodily weakness until early in the year 1913, when he suffered from what may have been a form of influenza. He said that it was the first time in his life that he had been sick and that he was "all played out." He was then seventy-five years old. He never again fully recovered his strength, so that his visits to the observatory thereafter were fewer and his last observations with the 40-inch telescope were made on May 13, 1914. During that summer he retired, on a pension, in accordance with the statutes of the University of Chicago. He was still given the opportunity to use the telescope, but did not avail himself of it, and after 1917 he did not often leave his home. Toward the end of February, 1921, his hip was broken from a fall, and from this, in his enfeebled condition, recovery was impossible. His death occurred a fortnight later.

In 1868 Mr. Burnham was married to Mary Cleland, who survives him with their three sons and three daughters, together with eight grandchildren.

A complete bibliography of Mr. Burnham's writings need not be given here. Summaries of his observations have been published in more than one place: thus Volume II of the *Publications of the Lick Observatory* (1894) gives the place of publication of his nineteen catalogues of new double stars, which had been printed chiefly in the *Monthly Notices* of the Royal Astronomical Society or in the *Astronomische Nachrichten;* it contains the complete details of the observations of the last six catalogues, covering stars β 1026 to β 1274, which were discovered and measured at the Lick Observatory. That volume also includes some measures of nebulae with the 36-inch refractor. Again, for convenience of reference, these original sources of publication were collected in Volume I of the *Publications of the Yerkes Observatory*, covering the history of the 1290 β stars. He increased his list of new double stars but little after this; in fact, he avoided them, so that the total number of β stars is 1340.

Six extensive lists of measures were published in the *Astronomische Nachrichten* between 1907 and 1911. Many of these are also contained in the volume on proper motion stars. At Mr. Burnham's request a final collection of measures was extracted from his notebooks by Professor Philip Fox and published in the *Astronomical Journal* in 1918.

After the publication of the *General Catalogue*, Mr. Burnham still maintained his manuscript edition, keeping it up to date so as to include the great numbers of new double stars discovered by Professor Hussey and by Professor Aitken in their systematic searches for such objects. When his health began to fail, Mr. Burnham passed this valuable MS catalogue on to Professor Eric Doolittle, who was carrying it forward at the time of his premature and lamented death in 1920. In accordance with a prior arrangement, it then went to Professor Aitken, under whose careful supervision it will be maintained and a new extension published.

Professor Burnham received the Gold Medal of the Royal Astronomical Society in 1894, and the Lalande Prize of the French Academy of Sciences was awarded to him in 1904. Yale College gave him the honorary degree of A.M. in 1878, and in 1915 he received the honorary degree of Sc.D. from Northwestern University.

YERKES OBSERVATORY
June 1921

THE RELATIONSHIP OF ABSOLUTE MAGNITUDE TO SPACE-VELOCITY[1]

By W. S. ADAMS, G. STRÖMBERG, and A. H. JOY

ABSTRACT

Variation of the velocity of stars with absolute magnitude.—A statistical study of the radial, tangential, and space-velocities of 1350 stars, mostly of types F, G, K, and M, shows a marked correlation with absolute magnitudes. The results are given in the form of equations and tables. The increase in average space-velocity for a decrease of one magnitude in brightness varies with the type, but is of the order of 3 km/sec. The greater homogeneity of the giant stars as a class and their comparative freedom from large individual motions are indicated by the results. As would be expected for a random distribution of velocities as to direction, the average radial velocities are about half the corresponding average space-velocities.

Frequency of the space-velocities of stars cannot be represented by a distribution according to Maxwell's law, for there is a large excess of stars with high velocities, but a type of *frequency function* based upon the assumption of a normal error-distribution in the logarithms of the velocities is found to represent the observations much more closely. The function is proportional to $e^{-h^2(\log v - \log \bar{v})^2}$, where $\bar{v}$ is the geometrical mean space-velocity.

In a communication published in 1915, Kapteyn and Adams[2] showed that the radial velocities of a considerable number of stars depended upon proper motion and mean parallax in such a way as to indicate that the brighter stars intrinsically move more slowly than the fainter stars. More extensive investigations by Adams and Strömberg[3] in 1917, and by Strömberg in 1918,[4] based upon 1300 stars of spectral types F, G, K, and M, gave similar results. For this purpose the absolute magnitudes derived by the spectroscopic method were used, and not only the radial velocities but also the components at right angles to the direction of the sun's apex were included in the discussion. An average change in radial velocity of about 1.3 km for each unit of absolute magnitude was indicated by these investigations.

The subject had already been discussed by Eddington[5] on the basis of stars with parallaxes determined by the trigonometric

[1] *Contributions from the Mount Wilson Observatory*, No. 210.

[2] *Mt. Wilson Communications*, No. 1; *Proceedings of the National Academy of Sciences*, **1**, 14, 1915.

[3] *Mt. Wilson Contr.*, No. 131; *Astrophysical Journal*, **45**, 293, 1917.

[4] *Mt. Wilson Contr.*, No. 144; *Astrophysical Journal*, **47**, 7, 1918.

[5] *Stellar Movements*, p. 52, 1914.

method, and some of the important consequences of such a relationship had been pointed out by Eddington, Russell, Halm, and others. Additional evidence will therefore be of especial value for investigations dealing with the motions of stars in our Galaxy and their connection with spectral type and mass.

Previous studies of the subject have been limited to components of the stellar motions either in the line of sight or at right angles to it. With the rapid accumulation of parallax observations, both trigonometric and spectroscopic, and a similar increase in determinations of radial velocities, we can extend the investigation to the actual motions of the stars in space. In the pages which follow we shall compute the space-motions of the stars in our list, derive the frequency-law representing their distribution, and study the relationship of these motions to absolute magnitude.

The velocity-components of a star relative to the sun are computed from the following equations:

$$x = V \cos \alpha \cos \delta - \frac{k}{\pi}(\mu_\alpha \sin \alpha \cos \delta + \mu_\delta \cos \alpha \sin \delta)$$

$$y = V \sin \alpha \cos \delta + \frac{k}{\pi}(\mu_\alpha \cos \alpha \cos \delta - \mu_\delta \sin \alpha \sin \delta)$$

$$z = V \sin \delta + \frac{k}{\pi}\mu_\delta \cos \delta$$

where x, y, and z are the velocity-components in the equatorial system, expressed in kilometers per second, V is the radial velocity relative to the sun, α and δ are right ascension and declination, μ_α and μ_δ the corresponding proper motions, π is the parallax, and k is a constant equal to 4.737 km/sec.

If x_0, y_0, and z_0 are the velocity-components of the sun referred to the centroid of a large number of stars, we can obtain the components x_1, y_1, and z_1 of the star relative to this centroid from the equations

$$x_1 = x + x_0, \qquad y_1 = y + y_0, \qquad z_1 = z + z_0.$$

The space-velocity v of the star, corrected for the sun's motion, is therefore

$$v = \sqrt{x_1^2 + y_1^2 + z_1^2}\,.$$

The parallaxes used in this investigation are those obtained by the spectroscopic method and published recently.[1] In a few instances where the parallaxes are large the trigonometric values have been used instead. The proper motions are from the *Catalogue* of Boss or *Cincinnati Publications*, No. 18, with the addition

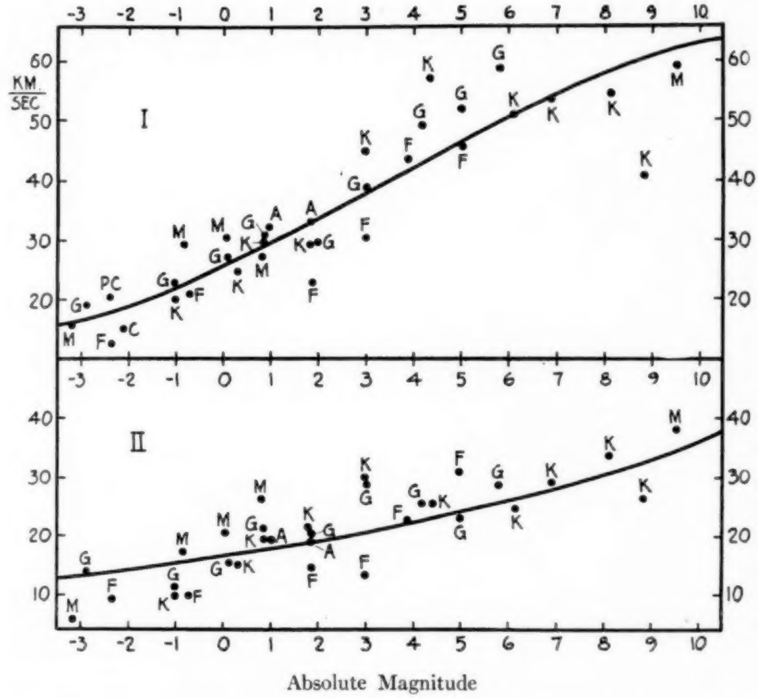

FIG. 1.—Curves showing the relationship of space-velocity to absolute magnitude for the various spectral types.

Curve I. Abscissae, average absolute magnitudes; ordinates, geometrical means of space-velocities. The continuous curve alone is shown.

Curve II. Abscissae, average absolute magnitudes; ordinates, average radial velocities.

of a few values determined by Kapteyn, Roy, and others. The radial velocities have been taken from Campbell's list in *Lick Observatory Bulletin*, No. 229, from the Mount Wilson lists published

[1] *Mt. Wilson Contr.*, No. 199; *Astrophysical Journal*, **53**, 13, 1921.

in *Contributions*, Nos. 79 and 105,[1] and from unpublished Mount Wilson results.

Since many of the stars observed at Mount Wilson were selected on the basis of small proper motion, it is evident that there may

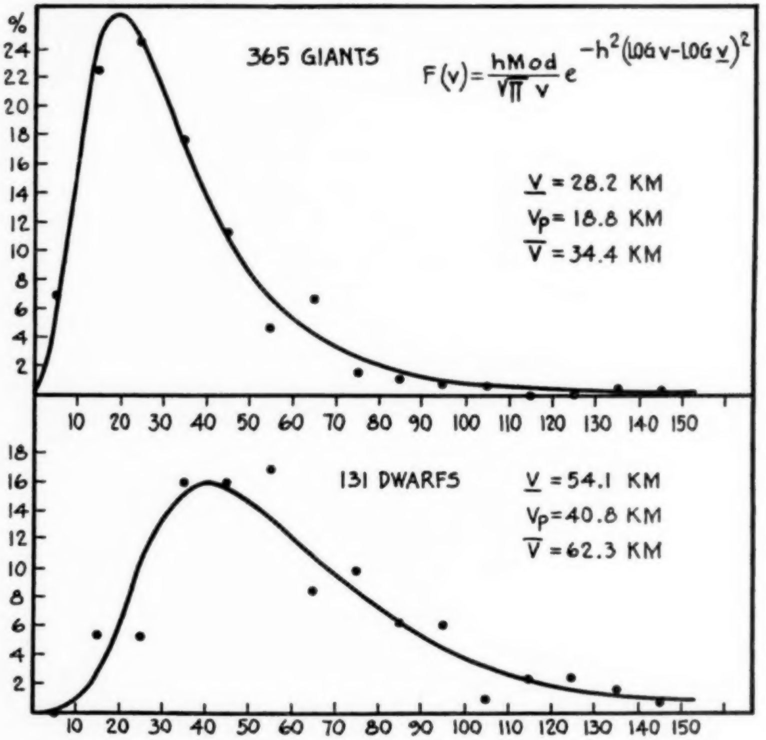

FIG. 2.—Frequency-curves for space-velocities of stars of spectral type K. Upper curve is based on 365 giant stars, lower curve on 131 dwarfs. Abscissae are space-velocities, ordinates are numbers of stars expressed in percentages of the whole within the limits of every 10 kilometers of space-velocity.

be too large a proportion of stars with velocity-components in a plane perpendicular to the line of sight, which are nearly equal in direction and amount to the sun's velocity projected on this plane. For this reason we have omitted from the discussion 101 stars,

[1] *Astrophysical Journal*, **39**, 341, 1914; **42**, 172, 1915.

this number being based on the distribution of proper motions given by Kapteyn in *Groningen Publications*, No. 30.

The detailed results based upon the values derived from the individual stars are given in Table I. The symbols in this table have the following meaning:

$\bar{M}$ = mean absolute magnitude
$\bar{v}$ = average space-velocity
$\bar{T}$ = average tangential velocity not corrected for the sun's motion. This is equal to $\overline{\left(\frac{\mu}{\pi}\right)}$
$\bar{\theta}$ = average radial velocity
$\underline{v}$ = geometrical mean of space velocities
$\underline{T}$ = geometrical mean of tangential velocities
r_1 and r_2 = probable deviations of log v and log T for a single star from the mean values log $\underline{v}$ and log $\underline{T}$.

All of these quantities have been computed from the individual values of v, T, log v, log T and V', the last being the radial velocity corrected for the sun's motion. The probable errors in log $\underline{v}$ and log $\underline{T}$ have been converted into corresponding errors in $\underline{v}$ and $\underline{T}$. The values used for the solar motion are those derived in 1917.[1]

$$A_0 = 270^\circ\!.9; \quad D_0 = +29^\circ\!.2; \quad V_0 = 21.48 \text{ km/sec.}$$

The use of the values $A_0 = 270^\circ$, $D_0 = +30^\circ$, $V_0 = 20.0$ km would in general affect $\bar{v}$ and $\underline{v}$ by less than 0.1 km.

The values for the A-type stars in Table I are based almost wholly upon stars of types A7 to A9, and the uncertainty in the determination of the absolute magnitudes may be larger than for stars of the succeeding types. Occasionally stars with absolute magnitudes falling on the limit between two groups are included with half-weight in each of the groups. Thus among the A-type spectra a star of absolute magnitude +1.5 is accountable for the fractional values appearing in the column giving the number of stars.

The use of the geometrical mean in this investigation has been found desirable for two reasons. In the first place it is influenced much less than the arithmetical mean by the inclusion of stars of

[1] *Mt. Wilson Contr.*, No. 144; *Astrophysical Journal*, **47**, 7, 1918.

TABLE I

Absolute Magnitude and Space-Motion

M	No.	$\bar{M}$	$\bar{v}$	$\bar{T}$	θ	v	T	r_1	r_2
	Type A (A7 to A9)								
			km	km	km	km	km		
−0.1 to +1.5....	11.5	+1.0	34.2	23.2	19.3	32.2± 2.4	19.4±2.6	±0.108	±0.201
+1.5 to +2.5....	14.5	1.9	36.6	31.8	19.6	32.9± 2.6	24.7±3.6	0.129	0.244
	Type F (F0 to F9)*								
−3.5 to −1.5....	9	−2.3	16.5	20.3	9.5	12.3± 2.6	15.2± 3.0	±0.271	±0.258
−1.5 to +0.5....	21	−0.6	43.7	44.6	11.7	23.7	19.8	....	....
	20†	−0.7	32.2	33.7	9.8	20.9± 3.7	17.4± 3.9	0.338	0.427
+0.5 to +2.5....	54.5	+1.9	28.1	26.9	14.6	22.8± 1.4	18.5± 1.7	0.198	0.286
2.5 to 3.5....	90	3.0	52.4	49.1	22.3	34.1± 1.7	30.3± 1.8	0.198	0.241
	85.5‡	3.0	34.5	33.4	13.4	30.1± 1.1	26.8± 1.4	0.150	0.205
3.5 to 4.5....	109	3.9	77.4	73.3	29.0	51.5± 2.8	47.6± 2.6	0.246	0.246
	99.5‡	3.9	54.5	51.9	22.5	43.6± 2.1	40.3± 2.0	0.208	0.211
4.5 to 5.7....	8.5	5.0	143.5	125.4	74.4	76.9±20.6	67.1±17.8	0.336	0.331
	6.5‡	5.0	54.0	47.4	31.0	45.5±10.4	39.7± 9.6	0.249	0.266
	Type G (G0 to G9)								
−4.5 to −1.5....	22	−2.9	30.1	27.0	13.7	21.7± 2.9	17.9± 2.5	±0.271	±0.288
	20	−2.9	25.6	21.7	14.0	19.2± 2.5	15.4± 2.0	0.255	0.263
−1.5 to −0.5....	25.5	−1.0	27.8	25.9	11.0	22.7± 1.9	19.4± 2.0	0.187	0.222
−0.5 to +0.5....	62.5	+0.1	35.7	32.2	15.5	27.2± 1.5	22.4± 1.5	0.193	0.233
+0.5 to +1.5....	72.5	0.9	37.8	31.8	21.0	30.2± 1.5	23.6± 1.5	0.186	0.230
1.5 to 2.5....	35.5	1.9	35.9	29.0	19.8	29.6± 2.0	22.7± 1.8	0.181	0.210
2.5 to 3.5....	18	3.0	45.4	39.0	29.4	38.8± 3.2	33.2± 3.4	0.154	0.189
3.5 to 4.5....	42.5	4.2	59.7	57.2	25.6	49.4± 3.4	47.6± 3.2	0.198	0.189
4.5 to 5.5....	73.5	5.0	58.7	58.2	23.1	51.9± 2.4	48.6± 2.5	0.168	0.187
5.5 to 6.6....	51	5.8	88.3	88.0	34.0	61.9± 3.4	63.4± 3.4	0.174	0.166
	50§	5.8	70.3	70.7	28.7	58.6± 3.0	60.0± 2.9	0.157	0.148
	Type K (K0 to K9)								
−4.0 to 0.0....	14	−0.8	44.4	42.2	11.9	26.6± 5.5	21.8± 5.0	±0.333	±0.372
	12	−1.0	28.0	23.6	9.9	20.0± 3.2	15.7± 3.2	0.276	0.312
0.0 to +0.5....	74.5	+0.3	29.4	22.8	14.6	24.4± 1.2	14.8± 1.1	0.182	0.289
+0.5 to +1.5....	186.5	0.9	36.5	28.6	19.5	29.6± 1.0	20.4± 0.8	0.190	0.248
1.5 to 2.5....	47	1.9	34.6	24.6	21.1	29.4± 1.7	17.0± 1.5	0.173	0.269
2.5 to 3.5....	11.5	3.0	48.8	37.6	29.8	45.9± 3.6	29.4± 4.6	0.116	0.231
3.5 to 5.5....	12.5	4.4	62.6	53.5	25.8	57.2± 4.6	32.7± 5.1	0.125	0.242
5.5 to 6.5....	67.5	6.1	67.3	70.4	24.3	50.9± 2.5	59.8± 2.8	0.177	0.162
6.5 to 7.5....	36	6.9	59.3	56.4	29.3	53.5± 2.8	52.8± 2.2	0.136	0.105
7.5 to 8.5....	12.5	8.1	58.7	54.8	33.7	54.2± 5.0	50.9± 4.3	0.143	0.130
8.5 to 9.7....	8	8.8	47.3	41.0	26.5	40.6± 6.7	36.2± 5.4	0.202	0.182
	Type M (Ma to Md)								
−3.8 to −2.7....	2	−3.2	16.4	11.2	5.6	15.8± 3.4	11.0± 1.7	±0.131	±0.095
−1.5 to −0.5....	13.5	−0.8	36.4	33.1	17.4	29.6± 2.7	21.4± 4.8	0.142	0.350
−0.5 to +0.5....	67.5	+0.1	36.5	28.2	20.4	30.2± 1.5	19.5± 1.4	0.174	0.249
+0.5 to +1.5....	31	0.8	32.0	26.3	26.3	27.2± 1.8	21.3± 1.8	0.154	0.203
8.5 to 10.0....	7	9.5	61.3	53.9	38.3	58.8± 4.7	49.3± 6.2	0.093	0.143
10.0 to 13.4....	11	10.9	62.3	50.0	37.5	51.4± 7.6	41.2± 5.8	0.214	0.203

* *Cin.* 560 (Sp. = A8, $M = 2.8$, $v = 557$, $V' = +327$) included in F type.

† RR Lyrae ($v = 274$, $V' = -49$) omitted.

‡ Stars with $v > 180$ omitted. These appear to form a separate group.

§ *Cin.* 2019 ($v = 986$) omitted.

TABLE I—*Continued*

M	No.	$\bar{M}$	$\bar{v}$	$\bar{T}$	θ	$\underline{v}$	$\underline{T}$	r_1	r_2
	CEPHEID VARIABLES ‖								
			km	km	km	km	km		
−3.4 to −2.1....	7	−2.6	19.9	24.6	10.1	12.2 ± 3.5	21.2 ± 3.3	±0.327	±0.178
−2.0 to −1.1....	8	−1.8	20.5	21.9	12.4	17.8 ± 2.4	19.9 ± 2.5	0.166	0.156
Sum and means	15	−2.1	20.2	23.2	11.3	14.9 ± 2.3	20.5 ± 1.9	0.255	0.159
	PSEUDO-CEPHEIDS								
−4.2 to −2.8....	14	−3.6	28.2	24.4	9.8	21.4 ± 3.7	15.6 ± 3.2	±0.277	±0.334
−2.7 to −0.1....	14	−1.2	30.6	22.4	19.3	19.0 ± 4.1	12.9 ± 2.5	0.353	0.308
Sum and means	28	−2.4	29.4	23.4	14.6	20.1 ± 2.7	14.2 ± 2.0	0.309	0.315

‖ RR Lyrae (M = 0.3, v = 274 km, V' = −49) omitted.

exceptionally high velocities. In the second place, as will be shown later, the form of frequency-function representing most closely the observed distribution of the space-velocities is one in which the logarithm of the geometrical mean of the velocities is one of the fundamental constants. The conversion of the geometrical into the arithmetical mean may be made readily through multiplication by a factor dependent upon the dispersion in the logarithms of the space-velocity around the mean value. This has been found to be fairly constant for all the groups of stars.

Attention should be called to two points bearing on the results of Table I. The first is that of the effect of the selection of the stars. Reference has already been made to the omission of a number of stars of small proper motion in order to equalize the proportion of such stars. The effect of the selection of stars of large proper motion would no doubt have been very marked had all stars of the same spectral type been included in one group. The division according to absolute magnitude should, however, eliminate to a large extent the effect of selection, since most of the stars of large proper motion may be assumed to have been selected on account of their nearness and hence their low intrinsic brightness. A slight excess of rapidly moving stars of high luminosity might, however, be expected from a selection of stars of large proper motion, and this would tend to increase slightly the mean values of the giant stars in Table I.

A second point has to do with the stars of the very highest luminosity. This group contains some stars which have erroneously been estimated as too bright, and as there is a tendency toward an accumulation of such errors, the mean space-velocity and the mean tangential velocity are too large. In order to reduce this effect a special computation has been made for the very brightest G and K stars, omitting a few of the largest space-velocities. This result is given in the second line of the G and K subdivisions of Table I.

From the results of Table I it appears that the values of r_1, although somewhat larger for the brighter stars, do not show any very great range. Accordingly we may with sufficient accuracy adopt its mean value for all absolute magnitudes and spectral types. This value is

$$r_1 = 0.171 \pm 0.0033.$$

As will be shown in connection with the discussion of the frequency-law, the arithmetical mean of the space-velocities $\bar{v}$ and the most frequent velocity v_p can be computed from the geometrical mean by aid of the equations

$$\bar{v} = 10^{2.530\, r_1^2}\, \underline{v}, \quad v_p = 10^{-5.061\, r_1^2}\, \underline{v}.$$

Hence

$$\bar{v} = 1.186\, \underline{v}, \qquad v_p = 0.7113\, \underline{v}.$$

The values of $\bar{T}$ and $\underline{T}$ have been computed because of their importance in the derivation of mean parallaxes. Since

$$\bar{T} = k\,\overline{\left(\frac{\mu}{\pi}\right)},$$

we have approximately

$$\bar{\pi} = k\frac{\bar{\mu}}{\bar{T}}.$$

Furthermore

$$\underline{T} = k\,\underline{\left(\frac{\mu}{\pi}\right)} = k\frac{\underline{\mu}}{\underline{\pi}},$$

or

$$\underline{\pi} = k\frac{\underline{\mu}}{\underline{T}} = 10^{0.2(\bar{M}-m)-1}.$$

With the aid of the data given in Table I we can now compute the general relationship between absolute magnitude and velocity. The geometrical means of the space-velocities and the tangential velocities have been used, since these can be obtained with a higher degree of accuracy than the arithmetical means, but the average radial velocity has been used directly. The tangential velocities are not corrected for the solar motion.

Two different solutions have been made. In the first solution we assume for each type the relationship

$$\log \underline{v}=a+bM,$$

with similar expressions for $\log \underline{T}$ and $\log \theta$, and use these equations under the assumption that they are constant for each spectral

TABLE II

EQUATIONS CONNECTING ABSOLUTE MAGNITUDE WITH VELOCITY
FIRST SOLUTION: CONTINUOUS VARIATION

Type	$\log \underline{v}$	$\log \underline{T}$	$\log \theta$
F.........	$1.356+0.0553\,M$ $\pm 0.032 \pm 0.0111$	$1.248+0.0690\,M$ $\pm 0.033 \pm 0.0113$	$1.149+0.0257\,M$ $\pm 0.044 \pm 0.0154$
G.........	$1.415+0.0605\,M$ $\pm 0.008 \pm 0.0021$	$1.313+0.0768\,M$ $\pm 0.016 \pm 0.0043$	$1.214+0.0412\,M$ $\pm 0.021 \pm 0.0067$
K.........	$1.428+0.0410\,M$ $\pm 0.018 \pm 0.0038$	$1.224+0.0649\,M$ $\pm 0.030 \pm 0.0063$	$1.232+0.0338\,M$ $\pm 0.017 \pm 0.0036$
M........	$1.456+0.0302\,M$ $\pm 0.013 \pm 0.0026$	$1.300+0.0402\,M$ $\pm 0.019 \pm 0.0037$	$1.306+0.0282\,M$ $\pm 0.025 \pm 0.0049$

ALL TYPES

Space-Velocity $\log \underline{v}=+1.408+0.0632M-0.00240M^2$
$\pm 0.011 \pm 0.0067 \quad \pm 0.00085$

Tangential Velocity $\log \underline{T}=+1.256+0.0870M-0.00303M^2$
$\pm 0.017 \pm 0.0098 \quad \pm 0.00124$

Average Radial Velocity $\log \theta=+1.219+0.0321M+0.00020M^2$
$\pm 0.017 \pm 0.0103 \quad \pm 0.00131$

type over the entire range of absolute magnitude. In the second solution a discontinuity is assumed for the K and M stars at the point of division between the giants and dwarfs. The evidence

afforded by Table I appears on the whole to be favorable to such a discontinuity, since the velocities of the dwarf stars of these types show little if any variation with absolute magnitude.

TABLE III

SECOND SOLUTION: SEPARATE EQUATIONS FOR GIANTS AND DWARFS OF TYPES K AND M

Type	$\log v$	$\log T$	$\log \theta$
	$M<2.5$		
K	$1.398+0.0531\,M$ $\pm0.024\pm0.0206$	$1.232+0.0281\,M$ $\pm0.052\pm0.0446$	$1.164+0.1062\,M$ $\pm0.028\pm0.0239$
M	$1.455+0.0320\,M$ $\pm0.016\pm0.0213$	$1.298+0.0440\,M$ $\pm0.014\pm0.0191$	$1.304+0.1480\,M$ $\pm0.012\pm0.0165$
All (F to M)	$1.422+0.0282\,M$ $\pm0.010\pm0.0062$	$1.275+0.0417\,M$ $\pm0.013\pm0.0082$	$1.232+0.0113\,M$ $\pm0.022\pm0.0135$
	$M>2.5$		
K	1.710 ±0.012	1.700 ±0.012	1.431 ±0.014
M	1.734 ±0.031	1.645 ±0.040	1.578 ±0.012
All (F to M)	$1.640+0.0126\,M$ $\pm0.024\pm0.0042$	$1.549+0.0234\,M$ $\pm0.053\pm0.0089$	$1.271+0.0271\,M$ $\pm0.030\pm0.0051$

Tables II and III give the values of the coefficients and their probable errors in the equations of the form

$$\log v = a + bM$$

for the two solutions. Further, all the types are discussed together, and a term involving the square of M has then been included in the first solution, the form being

$$\log v = a + bM + cM^2$$

with similar expressions for the tangential and radial velocities.

From the equations of Tables II and III we obtain the values given in Table IV. The columns marked 1 and 2 under K, M,

TABLE IV

ABSOLUTE MAGNITUDE AND VELOCITY: COMPUTED VALUES

M	F	G	K 1	K 2	M 1	M 2	All Types 1	All Types 2
			$\underline{v}$ = GEOMETRICAL MEAN OF SPACE-VELOCITIES					
− 2	17.6	19.7	22.2	19.6	24.9	24.6	18.7	23.2
− 1	20.0	22.6	24.4	22.1	26.7	26.5	22.0	24.8
0	22.7	26.0	26.8	25.0	28.6	28.5	25.6	26.4
+ 1	25.8	29.9	29.4	28.2	30.6	30.7	29.4	28.2
2	29.3	34.4	32.4	31.9	32.8	33.0	33.5	30.1
3	33.3	39.4	35.6		......	......	37.7	47.6
4	37.8	45.4	39.1		......	......	41.9	49.0
5	42.8	52.2	43.0	51.3	......	......	46.1	50.5
6	......	60.0	47.2		......	......	50.2	52.0
7	......	......	51.9		......	......	54.1	53.5
8	......	......	57.0		49.9		57.5	55.1
9	......	......	......	......	53.5	54.2	60.5	56.6
+10	......	......	......	......	57.3		63.1	58.3
			$\underline{T}$ = GEOMETRICAL MEAN OF TANGENTIAL VELOCITIES					
− 2	12.9	14.4	12.4	15.0	16.6	16.2	11.8	15.6
− 1	15.1	17.2	14.4	16.0	18.2	18.0	14.7	17.1
0	17.7	20.6	16.8	17.1	20.0	19.9	18.0	18.8
+ 1	20.8	24.6	19.4	18.2	21.9	22.0	21.9	20.8
2	24.3	29.3	22.6	18.4	24.0	24.3	26.2	22.8
3	28.5	34.9	26.2		......	......	30.9	41.6
4	33.4	41.7	30.5		......	......	36.0	44.0
5	39.2	49.8	35.3	50.1	......	......	41.2	46.3
6	......	59.4	41.0		......	......	46.7	48.9
7	......	......	47.6		......	......	52.0	51.6
8	......	......	55.3		41.9		57.3	54.4
9	......	......	......		45.9	44.2	62.2	57.5
+10	......	......	......	......	50.4		66.5	60.7
			θ = AVERAGE RADIAL VELOCITY					
− 2	12.5	13.6	14.6	9.0	17.8	10.2	14.3	16.2
− 1	13.3	14.9	15.8	11.4	19.0	14.3	15.4	16.6
0	14.1	16.4	17.1	14.6	20.2	20.1	16.6	17.1
+ 1	15.0	18.0	18.4	18.6	21.6	28.3	17.8	17.5
2	15.8	19.8	20.0	23.8	23.0	39.8	19.2	18.0
3	16.8	21.8	21.5		......	......	20.8	22.5
4	17.9	23.9	23.3		......	......	22.4	23.9
5	19.0	26.3	25.2	27.0	......	......	24.2	25.5
6	......	28.9	27.2		......	......	26.2	27.2
7	......	......	29.4		......	......	28.4	28.9
8	......	......	31.8		34.0		30.8	30.8
9	......	......	......	......	36.3	37.8	33.4	32.7
+10	......	......	......	......	38.7		36.3	34.8

and All Types give the results derived from the two solutions, the first representing a continuous and the second a discontinuous variation.

The geometrical means of the space-velocities may be converted into arithmetical means by multiplication with the factor 1.186. The results are given in Table V, the values used being those derived from the first solution, based on the assumption of a continuous variation with absolute magnitude.

TABLE V

ARITHMETICAL MEAN OF SPACE-VELOCITY AND CORRECTED AVERAGE RADIAL VELOCITY

M	$\bar{v}$					θ				
	F	G	K	M	All	F	G	K	M	All
−2	20.9	23.4	26.3	29.5	22.2	11.9	12.9	13.9	16.9	13.6
−1	23.7	26.8	28.9	31.7	26.1	12.6	14.2	15.0	18.0	14.6
0	26.9	30.8	31.9	33.9	30.4	13.4	15.6	16.2	19.2	15.8
+1	30.6	35.5	34.9	36.3	34.9	14.2	17.1	17.5	20.5	16.9
2	34.7	40.8	38.4	38.9	39.7	15.0	18.8	19.0	21.8	18.2
3	39.5	46.8	42.2	......	44.7	16.0	20.7	20.4	......	19.8
4	44.8	53.8	46.4	......	49.7	17.0	22.7	22.1	......	21.3
5	50.8	61.9	51.0	......	54.7	18.0	25.0	23.9	......	23.0
6	......	71.2	56.0	......	59.5	......	27.5	25.8	......	24.9
7	......	......	61.6	......	64.2	......	......	27.9	......	27.0
8	......	......	67.6	59.2	68.2	......	......	30.2	32.3	29.3
9	......	......	......	63.5	71.8	......	......	......	34.5	31.7
+10	......	......	......	68.0	74.8	......	......	......	36.8	34.5

The average radial velocities θ are derived from stars north of $-30°$ declination with absolute magnitudes determined at Mount Wilson. Consequently, on account of the effect of stream-motion, the values of θ require a slight systematic correction if they are to be compared with values obtained from stars distributed over the whole sky. Allowance is made for the omission of stars in the area south of $-30°$ by multiplication with a factor 0.95 and the results of θ corrected in this way are also given in Table V.

The mean velocities for the giants and dwarfs separately have been computed for each spectral type and are given in Table VI.

It is of interest to compare the results for θ, the average radial velocity corrected for stream-motion, with those given by Campbell.[1] His values are:

F	G	K	M
14.4	15.9	16.9	17.1 km/sec.

The agreement is excellent if we assume the following mean absolute magnitudes for the stars of Campbell's list:

F, +1; G, 0; K, 0 to +1; M, −2.

These values of the mean absolute magnitudes for the stars observed by Campbell appear quite reasonable, except perhaps in

TABLE VI

SUMMARY OF RESULTS

		F	G	K	M
Giants	No.	152	216	320	114
$M<2.5$	$\overline{M}$	0.1	0.3	0.8	0.1
	$\underline{v}$	21.7	27.0	27.9	29.0
	v	25.7	32.0	33.1	34.4
	$\underline{T}$	18.0	21.7	18.2	20.0
	θ	14.1	17.4	18.2	21.4
Corrected	θ	13.4	16.5	17.3	20.3
Dwarfs	No.	191	184	148	18
$M>2.5$	$\overline{M}$	3.5	4.8	6.2	10.3
	$\underline{v}$	37.0	51.5	51.3	54.2
	v	43.9	61.1	60.8	64.3
	$\underline{T}$	33.6	49.3	50.1	44.2
	θ	18.7	25.8	27.0	37.8
Corrected	θ	17.8	24.5	25.6	35.9

the case of the stars of the M type. For these stars it seems probable that the systematic motions differ somewhat from those of other types, and hence our values of θ need a special correction in order to make them comparable with those obtained from stars distributed over the whole sky.

[1] *Lick Observatory Bulletin*, **6**, 125 (No. 196), 1911.

It has been shown by Campbell[1] that, if equal numbers of stars move in all directions, the average radial velocity should be exactly half the average space-velocity. This is strictly true only if stream-motion is neglected. A comparison of the results in Table V shows a remarkably close approach to this ratio among the giant stars. Thus for absolute magnitudes <2 we find for the ratio of space-velocity to radial velocity the values:

F	G	K	M	All Types
2.02	1.98	1.96	1.77	1.92

In view of the uncertainties introduced by possible differences in the systematic motions not allowed for in the determination of the average radial velocity, these results are rather surprising.

For the dwarf stars the relationship holds much less closely, especially in types F and G. These stars in general have very high space-velocities, and, as will be shown in connection with an investigation of the directions of stellar motions, they exhibit very marked differences from the giants in their systematic motions.

The variation of velocity with spectral type among the giant stars is fully confirmed by these results, the space-velocities showing it quite as well as the radial motions. The latter values, as already noted, are in good agreement with those of Campbell, except possibly for the M-type stars where the difference from those of type K is somewhat larger. Among the dwarf stars, the existence of such a relationship is very doubtful and there even appears to be a tendency toward a reversal of the effect found for the giants, the F and G stars moving more rapidly than the K and M stars. As already stated, the largest space-velocities are found among the F and G types. In view of the peculiar character of the motions of the rapidly moving dwarf stars it seems very doubtful if we can distinguish a relationship dependent upon absolute magnitude without a further great addition to our observational material.

The values of the radial velocities shown in Table I among the very brightest stars of spectral types F and K and the Cepheid variables are quite comparable with those of stars of type B, and suggest that there is little variation with spectral type among

[1] *Stellar Motions*, p. 215, 1914.

such stars. The material, however, is insufficient for a definite conclusion. The space-velocities of these stars cannot well be compared, since they are affected systematically by any accumulation of errors in their parallaxes or proper motions.

It is clear from the results of this investigation that the giant stars as a whole are much more homogeneous in character than the dwarf stars and show the effects of variation of velocity with absolute magnitude and spectral type more clearly. The latter show a much wider dispersion of velocities and are subject to systematic motions differing from those of the giant stars. This is perhaps to be expected if the masses of the giant and dwarf stars differ to any considerable extent.

THE FREQUENCY-LAW OF SPACE-VELOCITIES

It is of interest to compare the observed distribution of these space-velocities with that given by Maxwell's law. This assumes a

TABLE VII

COMPARISON OF FREQUENCY OF SPACE-VELOCITIES WITH THAT GIVEN BY MAXWELL'S LAW

v	F Type $M=2.5$ to 5.7		G Type $M=-1.5$ to $+2.5$		G Type $M=3.5$ to 6.6		K Type $M=-4.0$ to $+2.5$		K Type $M=5.5$ to 9.7		M Type $M=-3.8$ to $+1.5$	
	N_o	N_c	N_o	N_c	N_o	N_c	N_o	N_c	N_o	N_c	N_o	N_c
km/sec.												
0–20....	33	33	55	55	14	14	107	107	7	7	30	30
20–40....	90	90	101	101	39	39	154	154	28	28	57	57
40–60....	41	39	30	20	45	17	58	19	43	27	23	13
60–80....	16	5	17	1	33	2	30	0	24	11	7	1
>80....	47	0	12	0	49	0	16	0	29	2	6	0
Total..	227	167	215	177	180	72	365	280	131	75	123	101
h......	0.0352		0.0428		0.0351		0.0472		0.0263		0.0417	
Excess in per cent..	36		22		151		30		76		22	

random distribution in direction and contains a single constant, the modulus h, in the equation

$$F(v)dv=\frac{4h^3}{\sqrt{\pi}}e^{-h^2v^2}v^2\,dv\,.$$

Determining the value of h from the numbers of velocities between 0 and 20 km and 20 and 40 km, we obtain the results shown in Table VII. N_o and N_c denote the numbers of observed and computed velocities within the limits given.

This comparison at once shows a large excess in the number of observed high velocities over that required by Maxwell's law. It is especially marked among the dwarf stars and is probably due to the fact that a strong correlation exists among the three components of space-velocities, a large component along one axis being in general related to large components along the other two axes. Maxwell's function is based upon the assumption that the three components are independent of one another.

A type of frequency-function which represents much better the observed distribution of space-velocities is one based upon the assumption of a normal error-distribution of the logarithms of the velocities.[1] If we transform such a function into one expressing the frequencies of the velocities themselves instead of their logarithms, we obtain the form

$$F(v)dv = \frac{h \text{ Mod}}{\sqrt{\pi} v} e^{-h^2 (\log v - A)^2} dv ,$$

the logarithms here having the base 10.

This function contains two constants, h and A, the former of which is related to the probable error in the difference $\log v - A$ by the equation

$$r = \frac{0.6745}{\sqrt{2}\, h} .$$

The quantity A is the algebraic mean of the logarithms of the space-velocities, or,

$$A = \overline{\log v} = \log \underline{v} ,$$

where $\underline{v}$ represents the geometrical mean of the velocities.

[1] Another type of frequency-function, involving only one constant, which represents the distribution nearly as well as the logarithmic one, is

$$F(v)dv = \frac{h^3}{2} v^2 e^{-hv} dv .$$

This gives

$$\bar{v} = \frac{3}{h} , \quad v_p = \frac{2}{h} .$$

The arithmetical mean $\bar{v}$ and the most probable value v_p are related to $\underline{v}$, h, and r by the expressions

$$\log \bar{v} = \log \underline{v} + \frac{1}{4h^2 \,\mathrm{Mod}} = \log \underline{v} + 2.530\, r^2,$$

$$\log v_p = \log \underline{v} - \frac{1}{2h^2 \,\mathrm{Mod}} = \log \underline{v} - 5.061\, r^2.$$

Hence

$$v_p < \underline{v} < \bar{v}.$$

Table VIII shows how this frequency-function represents the observed velocities. The stars have been grouped according to

TABLE VIII

FREQUENCY OF SPACE-VELOCITIES ACCORDING TO LOGARITHMIC FUNCTION

v	F Type $M=2.5$ to 5.7		G Type $M=-1.5$ to $+2.5$		G Type $M=3.5$ to 6.6		K Type $M=-3.5$ to $+2.5$		K Type $M=5.5$ to 9.7		M Type $M=-3.8$ to $+1.5$	
	N_o	N_c	N_o	N_c	N_o	N_c	N_o	N_c	N_o	N_c	N_o	N_c
km												
0– 10	3	3.7	16	8.4	3	0.7	25	18.6	0	0.1	6	3.8
10– 20	30	31.7	39	49.2	11	9.9	82	88.7	7	3.8	23.5	26.8
20– 30	53	44.9	59	54.0	17	20.7	89	89.4	7	13.5	37.0	31.8
30– 40	37	38.7	42	38.7	21.5	24.9	65	62.5	21	19.9	20.5	23.1
40– 50	29	28.2	17	24.5	22.5	23.4	41	39.4	21	20.5	14	14.6
50– 60	12	19.2	13	15.0	23	20.1	17	24.1	22	17.8	9	8.9
60– 70	10	12.9	7	9.1	18.5	16.6	24	14.9	11	14.3	5	5.3
70– 80	6	8.7	10	5.8	14.5	13.1	6	9.2	13	10.8	2	3.2
80– 90	9	5.8	1	3.4	16	10.6	4	6.2	8	8.2	1	1.9
90–100	8	3.9	1	2.4	10	8.1	3	3.6	8	5.9	2	1.3
100–110	2	2.7	2	1.5	7	6.3	2	2.6	1	4.3	0	0.7
110–120	2	1.8	0	0.8	4	5.1	0	1.8	3	3.1	0	0.6
120–130	1	1.3	2	0.7	3	4.1	0	1.1	3	2.4	0	0.3
130–140	1	1.1	1	0.4	2	3.1	1	0.7	2	1.6	0	0.2
140–150	2	0.7	4	0.2	1.5	2.5	1	0.7	1	1.3	1	0.1
150–200	9	2.5										
200–250	0	2.6										
250–300	2	3.4										
300–350	2.5	3.4	1	0.9	5.5	10.8	5	1.5	3	3.5	2	0.4
350–400	2.5	3.0										
400–450	2	2.2										
450–500	1	1.7										
>500	3	2.9										
Totals	227	227.0	215	215.0	180	180.0	365	365.0	131	131.0	123	123.0

spectral type and absolute magnitude. In the case of the F-type stars it was found necessary to use the sum of two frequency-functions, since there exists in this type a special group of stars of absolute magnitudes between 3 and 5 with velocities ranging from 160 to nearly 1000 km.

The constants of the frequency-function $\underline{v}$ and r, together with the arithmetical mean $\bar{v}$ and the most probable velocity v_p, are given in Table IX.

TABLE IX

CONSTANTS OF LOGARITHMIC FREQUENCY-FUNCTION

Type	m	$\underline{v}$	r	v_p	$\bar{v}$	No.
		km		km	km	
F..........	+2.5 to +5.7	35.6	0.178	24.6	42.8	207
F..........	+2.5 to +5.7	346.0	0.104	306.0	368.0	20
G..........	−1.5 to +2.5	29.1	0.178	20.1	35.0	215
G..........	+3.5 to +6.6	55.0	0.190	36.2	67.7	180
K..........	−3.5 to +2.5	28.2	0.185	18.8	34.4	365
K..........	+5.5 to +9.7	54.1	0.156	40.8	62.3	131
M..........	−3.8 to +1.5	29.7	0.171	21.1	35.2	123

The marked difference in space-velocity between the giant and dwarf stars is shown clearly in the results of Table IX.

The principal results of this investigation may be summarized as follows:

1. The average space-velocities of the stars of types F, G, K, and M vary with absolute magnitude to a marked degree. The increase of velocity with decreasing brightness is most regular among the giant stars. Among the fainter dwarf stars, if the effect is present it is concealed in large measure by the individual peculiarities of motion.

2. The tangential and radial velocities show results similar to those obtained from the space-velocities.

3. The increase in average space-velocity is about 3 km/sec. for a decrease of one magnitude in brightness. In the radial velocity the increase is 1.2 km/sec.

4. The average space-velocity of the giant stars is very nearly twice the average radial velocity, a result which would follow strictly if equal numbers of stars moved in all directions.

5. The variation of velocity with spectral type is well marked among the giant stars, but is much less certain among the dwarfs.

6. The very brightest stars intrinsically of all the spectral types appear to have nearly equal radial velocities.

7. The greater homogeneity of the giant stars as a class and their pamocrative freedom from large individual motions are

indicated by these results. The dwarf stars show a very wide dispersion in motions.

8. The frequency of the space-velocities cannot be represented adequately by a distribution according to Maxwell's law, there being a large excess of high velocities. A type of frequency-function based upon the assumption of a normal error-distribution in the logarithms of the velocities is found to satisfy the observations in a much better way.

MOUNT WILSON OBSERVATORY
April 1921

THE ELECTRIC FURNACE SPECTRUM OF SCANDIUM[1]

By ARTHUR S. KING

ABSTRACT

Spectrum of scandium in arc and in electric furnace, λ *3015 to 6559 A.*—A highly purified sample of scandia, prepared by Sir William Crookes, was placed in a graphite boat in the tube-resistance furnace operated as usual at low pressures. The relative *intensities* of 257 lines on grating spectrograms, some obtained with the furnace at a temperature of 2000, 2250, or 2600° C. and others with a carbon arc containing scandia, are recorded in Table I, together with the *classification* of each according to its behavior in furnace and arc. About 25 lines enhanced in the arc and 150 lines enhanced in the furnace are indicated by E and A respectively. The 29 arc-flame lines are all A lines but are scattered through temperature classes I to III; they cannot be due to oxidation, since none took place in the furnace. On the other hand, since the band lines are practically absent from furnace spectra, they are probably due to the oxide. These results are of great interest in *relation to solar and sun-spot spectra.* The A lines and low temperature furnace lines are absent or very weak in the solar spectrum but are prominent in the sun-spot spectrum.

Zeeman effect for scandium lines in sun-spot spectra is large and apparently uniform. Laboratory observations are lacking at present.

Observations as to chemical properties of scandium.—The scandia fused in the furnace seemed to form a carbide with the graphite boat. The shining black residue turned to a gray-brown powder after exposure to the air, presumably from the reabsorption of oxygen.

Scandium occurs in small amounts in a number of the rare-earth minerals, but its extraction in a fair degree of purity is an exceedingly difficult matter, and, as its separation has not been stimulated by any commercial application, the substance is extremely scarce. The late Sir William Crookes[2] described his examination of a large number of minerals in a search for those containing scandium. Wiikite, found in Finland, proved to contain over 1 per cent of scandium oxide (scandia) and was used by him to furnish pure scandia. The final difficulty in the tedious process of separation was the elimination of the closely related elements yttrium and ytterbium.

The spectrum of scandium is especially interesting on account of the peculiar intensity relations, to be discussed presently, among the scandium lines in the solar spectrum. An examination of the behavior of the lines at various temperatures of the electric

[1] *Contributions from the Mount Wilson Observatory,* No. 211.

[2] *Philosophical Transactions,* A, **209,** 15, 1909.

furnace was therefore desirable, in order to determine the part which temperature may play in bringing about the peculiarities observed.

Through the good offices of Professor Fowler, I have been able recently to obtain a sufficient amount of highly purified scandia to carry out the examination of the furnace spectrum. This scandia was prepared by Sir William Crookes. His former assistant, Mr. J. H. Gardiner, very kindly placed the material at my disposal through Professor Fowler.

In the earlier solar investigations in which the spot spectrum was studied it was noted that certain lines intensified in the spot coincided with lines in the scandium arc, while other strong arc lines were absent in the spot though present in the solar spectrum and prominent in the chromosphere. Fowler[1] explained this phenomenon by producing, besides the regular arc spectrum, the spectrum of the arc in hydrogen, which approaches that of the spark, and the spectrum of the flame of the arc, in which certain lines are given a high intensity. His results showed clearly that the lines prominent in the spot spectrum are those given by the flame of the arc and probably result from a reduced temperature in the spot, while the scandium lines missing from the spot spectrum are enhanced lines, though present often with considerable intensity in the arc. The occurrence of three distinct types, enhanced, arc, and flame lines, in the scandium arc is a feature shown by some other elements, notably titanium, the low-temperature lines of which have been selected by the furnace,[2] while many lines prominent in the arc are much enhanced in the spark.

EXPERIMENTAL METHOD

The tube-resistance furnace was operated as usual, the chamber being pumped out to a few millimeters' pressure. To avoid scattering of the scandia in the tube, the powder was put in a small combustion boat of graphite, which was placed at the middle of the furnace-tube. The image of the space above this boat was focused on the slit of the spectrograph. The scandia fused in the boat,

[1] *Ibid.*, A, **209**, 47, 1909.

[2] *Mt. Wilson Contr.*, No. 76; *Astrophysical Journal*, **39**, 139, 1914.

entering to some extent into the graphite as if a carbide were formed. The fused residue was a shining black when removed from the tube, but after exposure to the air for some time turned to a gray-brown powder, presumably from the reabsorption of oxygen.

The spectrograms were made with a 15-foot concave-grating spectrograph, the second order with a dispersion of 1 mm = 1.86 A being used for the ultra-violet, and the first order for the region from λ 3800 into the red.

EXPLANATION OF THE TABLE

The method followed in the study of the furnace spectrum of scandium was the usual one of selecting three temperature stages, the lowest of which gives a small number of distinctive lines. These lines are of two kinds, some remaining practically unchanged or even losing in strength as the temperature is increased, while others strengthen with rising temperature and frequently show wide reversals. These two kinds of lines are given in Table I as belonging to Class I and Class II respectively.

A higher stage, designated as medium temperature, brings out a large number of lines, placed as a rule in Class III, while a still higher temperature gives the lines of Class IV. Still other lines, which are absent or very faint in the high-temperature furnace, go into Class V. The furnace temperatures required to bring out lines of these various classes differ with different elements. For scandium the three stages selected were 1900° to 2000° C. for the low, 2250° for the medium, and 2600° for the high temperature.

In Table I, the wave-lengths in the first column are those of Exner and Haschek[1] for the arc spectrum. The arc wave-lengths measured by Fowler[2] cover the visible spectrum beginning at λ 3934 and include some lines not measured by Exner and Haschek. The wave-lengths, according to Fowler, of such lines are denoted by a dagger. The enhanced lines of scandium form an important class. I have taken Fowler's selection of these and designated them by "E" after the class-number. Finally, a large proportion

[1] *Spektren der Elemente bei normalem Druck*, Leipzig, 1911.

[2] *Loc. cit.*

TABLE I

TEMPERATURE CLASSIFICATION OF SCANDIUM LINES

λ Exner and Haschek	Arc Intensity	Furnace Intensities			Class
		High Temperature	Medium Temperature	Low Temperature	
3015.50*....	8	5r	3	2	II
3019.48*....	10	5r	3	2	II
3030.87*....	3	2	2	1	II
3040.03*....	....	1	1	....	III A
3056.37.....	1	1	....	....	IV
3251.43.....	1	1	....	....	IV
3255.81.....	6	15r	10r	10	I A
3270.08.....	15	40R	30R	15	II A
3273.79.....	20	50R	50R	20	II A
3333.67.....	1	5	2	....	III A
3343.42*....	1	4	1	....	III A
3349.36.....	1	4	1	....	III A
3351.34.....	1	3	....	....	IV A
3352.19.....	2	1	....	....	IV
3353.90.....	10	8	8	1	III
3359.84.....	6	6	4	....	III
3361.48.....	6	6	4	....	III
3362.12.....	6	6	4	....	III
3365.01.....	1	5	2	....	III A
3369.13.....	8	8	6	....	III
3372.33.....	12	10	10	1	III
3386.02.....	1	2	tr	....	IV A
3416.80.....	2	4	2	....	III A
3429.34.....	3	8	5	....	III A
3429.61.....	3	8	5	....	III A
3431.50.....	3	8	6	....	III A
3435.68.....	5	12	10	....	III A
3439.55.....	1	3	1	....	III A
3440.31.....	1	5	2	....	III A
3444.13.....	1	4	1	....	III A
3448.62.....	1	4	1	....	III A
3457.56.....	3	8	5	....	III A
3462.30.....	2	5	3	....	III A
3469.76.....	1	4	2	....	III A
3469.84.....	1	3	1	....	III A
3499.05.....	2	6	4	....	III A
3535.89.....	6	7	6	....	III
3558.69*....	7	8	8	3	II
3567.89.....	6	6	4	2	II
3572.73.....	20	20	20	10	II
3576.53.....	8	10	10	4	II
3581.11.....	7	8	10	4	II
3589.81.....	6	6	5	2	II
3590.67.....	6	6	5	2	II
3614.00.....	30	25	25	15	II
3630.93.....	12	12	15	8	II
3635.45*....	1	?	1	....	III
3642.99.....	25	20	20	7	II
3645.50.....	10	15	8	2	III
3647.05.....	1	4	1	....	III A

TABLE I—*Continued*

λ Exner and Haschek	Arc Intensity	Furnace Intensities			Class
		High Temperature	Medium Temperature	Low Temperature	
3652.01.....	7	8	6	2	III
3657.75.....	tr	3	1	....	III A
3666.68.....	2	3	1	....	III
3895.11.....	1	2	tr	....	IV A
3907.69.....	75	75R	60	15	II
3912.03.....	100	100R	80	15	II
3915.09.....	tr	1	....	....	IV A
3933.59.....	20	25r	15	10	II
3952.43.....	1	2	....	....	IV A
3996.79.....	30	30r	20	20	I
4014.68.....	5	3	tr	....	IV E
4020.60.....	75	75R	60	40	II
4023.36*....	2	5?	1	....	III A
4023.88.....	100	100R	80	40	II
4031.51.....	2	5	3	....	III A
4034.35.....	2	4	1	....	III A
4036.98.....	2	1	....	....	IV
4043.97.....	2	6	3	....	III A
4046.64.....	2	7	3	....	III A
4047.98.....	25	25r	25	20	I
4050.09.....	2	5	2	....	III A
4052.00......	1	2	tr	....	IV A
4054.71.....	35	40R	40	40	I
4056.72.....	4	8	4	....	III A
4067.15.....	2	6	3	....	III A
4075.13.....	2	7	3	....	III A
4078.70.....	2	6	3	....	III A
4082.60.....	40	30R	40	50	I
4086.15.....	1	3	1	....	III A
4086.80.....	4	5	3	....	III
4087.28.....	8	7	5	....	III
4095.03.....	1	4	1	....	III A
4133.10.....	8	8	7	....	III
4140.42.....	10	8	7	....	III
4152.51.....	12	10	8	....	III
4165.39.....	12	10	10	....	III
4171.92.....	1	3	2	....	III A
4218.43.....	2	6	2	....	III A
4219.90.....	2	7	2	....	III A
4222.07†....	1	3	tr	....	IV A
4225.76*....	2	5?	2	....	III A
4232.13.....	2	5	1	....	IV A
4233.83.....	3	8	2	....	III A
4237.96.....	2	3	tr	....	IV
4238.21.....	6	10	5	....	III
4239.72.....	2	5	1	....	IV A
4246.27.....	1	5	1	....	IV A
4247.02.....	75	10	10	....	III E
4259.86†....	1	1	....	....	IV
4283.71.....	2	6	1	....	IV A
4286.71.....	1	5	tr	....	IV A
4294.94.....	10	2	....	....	IV E

TABLE I—*Continued*

λ Exner and Haschek	Arc Intensity	Furnace Intensities			Class
		High Temperature	Medium Temperature	Low Temperature	
4305.89.....	10	2	....	....	IV E
4314.31.....	100	10	5	....	III E
4320.98.....	75	9	4	....	III E
4325.22.....	50	8	3	....	III E
4348.67.....	tr	2	....	....	IV A
4352.28.....	tr	2	....	....	IV A
4354.79.....	8	....	....	....	V E
4358.85.....	4	10	3	....	III A
4359.25.....	2	4	1	....	III A
4359.81.....	1	2	tr	....	IV A
4365.08.....	1	3	2	....	III A
4374.69.....	60	8	3	....	III E
4375.32.....	1	4	1	....	III A
4381.41.....	tr	3	1	....	III A
4384.98*....	8	?	?	....	IV E ?
4389.76.....	2	6	1	....	IV A
4400.63.....	50	7	2	....	III E
4415.78.....	40	7	2	....	III E
4431.52.....	4	....	....	....	V E
4542.74.....	2	8	3	....	III A
4544.86*....	5	10	4	....	III A
4557.45.....	5	7	2	....	III
4574.20.....	6	8	3	....	III
4579.13.....	1	6	1	....	IV A
4593.15.....	3	3	tr	....	IV
4598.32.....	1	3	tr	....	IV A
4598.63.....	3	5	1	....	IV
4604.89.....	(1)	6	1	....	IV A
4609.69.....	(0)	6	2	....	III A
4610.15.....	(0)	4	1	....	III A
4670.60*....	15	?	....	....	IV E
4709.51.....	5	4	2	....	III
4721.00.....	(0)	6	3	....	III A
4728.98.....	5	8	8	tr	III
4729.40.....	30	20	25	3	III
4732.48*....	2	5?	?	....	IV A
4734.30.....	15	15	20	4	III
4737.83.....	20	18	20	6	III
4741.23.....	30	30	30	8	III
4744.03.....	40	40	40	10	III
4746.31.....	(1)	3	tr	....	IV A
4749.15.....	(1)	2	....	....	IV A
4753.34.....	15	20	60	150	I A
4759.15.....	(1)	4	1	....	III A
4763.30.....	(3)	7	1	....	IV A
4771.67.....	(1)	2	....	....	IV A
4779.59.....	20	25	80	200	I A
4791.74.....	4	20	20	20	I A
4793.10.....	(0)	tr	....	....	IV
4827.48.....	4	12	3	....	III A
4833.86.....	2	10	3	....	III A
4839.63.....	10	20	8	....	III A

TABLE I—*Continued*

λ Exner and Haschek	Arc Intensity	Furnace Intensities			Class
		High Temperature	Medium Temperature	Low Temperature	
4840.65	2	6	tr	....	IV A
4841.04*	tr	4	2?	....	III A
4847.88	3	10	3	....	III A
4852.88	2	10	3	....	III A
4859.35†	(0)	2	....	....	IV A
4878.32	(2)	4	tr	....	IV
4880.93	(1)	5	tr	....	IV A
4890.54	(1)	3	....	....	IV A
4893.20	(1)	4	tr	....	IV A
4906.87	(2)	15	3	....	III A
4909.95	3	25	6	....	III A
4923.03	(2)	20	5	....	III A
4934.30*	(1)	5?	1	....	IV A
4935.95	(1)	4	1	....	III A
4941.51	2	10	3	....	III A
4951.46†	(1)	7	1	....	IV A
4951.84	(1)	6	1	....	IV A
4954.22	5	30	7	....	III A
4973.84	3	12	4	....	III A
4980.51	4	15	4	....	III A
4983.60*	2	20	3	....	III A
4992.14	5	15	3	....	III A
4995.24	(1)	7	1	....	IV A
5018.56	(2)	8	2	....	III A
5020.31	(2)	3	....	....	IV
5021.70	(2)	5	....	....	IV A
5031.23	10	1	....	....	V E
5032.90	(1)	2	....	....	IV
5064.51	10	15	5	tr	III
5069.05	2	2	1	....	III
5070.42	40	30	10	1	III
5076.00	10	10	4	tr	III
5081.80	125	40	40	20	II
5083.93	80	30	30	12	II
5085.75	40	15	15	6	II
5087.18*	40	?	15	6	II
5087.32*	10	?	1	....	IV ?
5090.10	8	5	1	....	IV
5092.63*	1	3?	....	....	IV A?
5096.92	30	15	4	1	III
5099.41*	40	30?	8	2	III
5101.30*	20	25?	5	1	III
5109.25*	3	?	1	....	III
5113.05*	4	?	1	....	III
5116.89*	10	?	2	....	III
5210.70	20	15	1	....	IV
5211.50	(0)	2	....	....	IV A
5219.06†	(0)	1	....	....	IV
5219.79	10	10	1	....	IV
5240.00	2	1	....	....	IV E
5258.51	15	15	1	....	IV
5285.15	tr	2	....	....	IV A

TABLE I—*Continued*

λ Exner and Haschek	Arc Intensity	Furnace Intensities			Class
		High Temperature	Medium Temperature	Low Temperature	
5285.91	10	10	1	....	IV
5302.15	2	60	20	12	II A
5314.91†	tr	2	....	....	IV A
5315.77†	tr	2	....	....	IV A
5318.53	1	tr	....	....	IV E
5323.26†	tr	3	....	....	IV A
5323.94†	(0)	tr	....	....	IV
5331.95	3	10	1	....	IV A
5334.40	tr	1	....	....	IV A
5339.56	8	12	2	....	IV
5341.20	8	12	2	....	IV
5343.13	10	50	40	30	II A
5349.49	50	20	5	....	III
5349.88	15	70	50	40	II A
5350.45	4	10	1	....	IV A
5355.92	8	10	2	....	III
5356.26	60	30	6	1	III
5375.52	20	20	4	....	III
5392.28	30	30	5	tr	III
5416.32	2	2	1	....	III
5425.76	2	3	1	....	III
5429.65	2	3	1	....	III
5433.45	2	2	1	....	III
5438.50	1	2	1	....	III A
5439.24	2	3	1	....	III
5442.80	2	3	1	....	III
5446.41	20	20	4	....	III
5447.66†	1	1	....	....	IV
5451.55	6	12	2	....	III A
5455.50	1	2	1	....	III A
5465.44	tr	1	....	....	IV A
5468.63	2	2	1	....	III
5472.42	3	3	1	....	III
5482.20	100	40	15	3	III
5484.80	80	30	10	2	III
5514.41	80	30	10	2	III
5520.71	100	40	15	3	III
5527.04	10	....	....	....	V E
5541.19	3	3	....	....	IV
5546.63†	1	1	....	....	IV
5565.10	4	10	2	....	III A
5591.55	15	30	3	....	IV A
5604.35	1	1	....	....	IV
5624.08†	1	1	....	....	IV
5631.23*	1	3?	....	....	IV A?
5641.18	2	....	....	....	V E
5646.58	4	5	1	....	IV
5647.78†	1	2	....	....	IV A
5649.75	6	10	1	....	IV
5658.13	15	tr	....	....	V E
5658.56	2	....	....	....	V E
5661.86†	tr	1	....	....	IV A

TABLE I—*Continued*

λ Exner and Haschek	Arc Intensity	Furnace Intensities: High Temperature	Furnace Intensities: Medium Temperature	Furnace Intensities: Low Temperature	Class
5667.38	2	....	....	....	V E
5669.26	2	2	....	....	IV E
5672.05	200	200	200	60	II
5684.41	8	tr	....	....	V E
5687.06	150	125	125	40	II
5691.55†	(0)	tr	....	....	IV
5700.35	100	100	100	30	II
5708.81	15	100	10	3	III A
5711.99	100	200	150	20	III A
5717.51	15	100	10	3	III A
5721.20†	tr	tr	....	....	IV
5724.30	15	100	10	3	III A
5739.53†	2	4	tr	....	IV A
5741.56†	1	5	1	....	IV A
5894.83†	(2)	3	tr	....	IV
5919.21†	tr	5	tr	....	IV A
5961.65†	4	4	tr	....	IV
5969.37†	3	3	tr	....	IV
5988.71	20	30	3	....	IV
6021.92†	(0)	1	....	....	IV
6026.37	15	20	2	....	IV
6146.50†	(3)	20	2	....	IV A
6193.94†	(3)	100	100	40	II A
6198.69	(2)	10	1	....	IV A
6210.85	200	500	500	800	I A
6239.64*	20	500	500	200	II A
6240.00	100	200	200	500	I A
6245.83	8	tr	....	....	V E
6250.15	15	20	1	....	IV
6259.20	100	300	300	400	I A
6262.50	10	15	1	....	IV
6273.37†	(0)	tr	....	....	IV
6276.52	15	300	200	100	II A
6284.66†	5	5	tr	....	IV
6293.30	(2)	4	tr	....	IV
6298.00†	(0)	tr	....	....	IV
6305.94	400	500	600	1500	I A
6306.26*	20	100	100	100	I A
6332.48	(1)	2	tr	....	IV
6345.06	5	100	75	50	II A
6379.07	40	300	300	300	I A
6396.62	2	2	tr	....	IV
6403.35	2	2	....	....	IV
6408.67	(1)	1	....	....	IV
6413.57	50	400	400	400	I A
6448.42†	1	100	50	20	II A
6486.56†*	(0)	2 ?	....	....	IV A ?
6525.84†*	(3)	1 ?	....	....	IV ?
6558.28†*	15	3 ?	tr	....	IV

λ	Remarks
3015–3040.	Weakened at high temperature through incipient absorption and appear readily in absorption when continuous ground is present. λ 3040 is faint in arc, given intensity 1 by Exner and Haschek.
3343.	Probably double.
3559.	Blend Ti. Probable intensity of Ti line subtracted.
3635.	Blend with impurity line at high temperature.
4023.36.	Blend with λ 4023.88 at high temperature.
4226.	Blend with impurity line at high temperature.
4385.	Blend V. Very faint in furnace.
4545.	Blend Ti. Probable intensity of Ti line subtracted.
4671.	Blend V. Faint if present in furnace.
4732.	Disturbed by carbon.
4841.	Blend Ti at medium temperature.
4934.	Slightly affected by Ba blend
4984.	Probably double.
5087–5117.	Disturbed by carbon.
5631.	Disturbed by carbon.
6239.64.	Measured in low-temperature furnace.
6306.26.	Measured in low-temperature furnace.
6487, 6526, 6558.	Disturbed at high temperature by band lines probably due to carbon.

of the scandium lines are found to be relatively strong in the furnace, at least at high temperatures, as compared with the arc. These are indicated by "A" after the class-number. The large number of these lines results in a measure from the choice of standard intensities in the arc and furnace spectra, but is chiefly due to the fact that there is less contrast among the scandium lines in the furnace than in the arc. Consequently faint lines often come out distinctly in the furnace, and if such lines were given the same intensity in arc and furnace very high values would have to be assigned to the stronger lines in the arc. The general meaning of "A" after the class-number then is that the furnace conditions are relatively favorable for the line in question

The arc intensities entered in the second column were estimated from photographs taken with scandia in the carbon arc. Not desiring to use much scandia for a minor part of the investigation, I made but one set of arc spectrograms, consequently there was no variation of the arc conditions. Some of the fainter arc lines measured by Exner and Haschek and by Fowler did not show in my arc spectra, usually on account of disturbance by the prominent

flutings. In such cases the arc values, usually from Fowler's list, are entered in parentheses.

An asterisk adjacent to the wave-length in Table I refers to a note among the "Remarks" at the end of the table. When a trace only of a line is to be seen, the intensity is denoted by "tr."

DISCUSSION

1. *Noteworthy lines of scandium.*—The temperature of approximately 2000°, designated as "low" for scandium, is about the same as has been employed as a medium temperature for several elements previously studied, so that the occurrence of low-temperature lines at wave-lengths as short as λ 3000 presents no anomaly. λ 2980.87 is probably to be classed with λλ 3015.50 and 3019.48, as it appeared at 2000°, but it is too near the end of the plates to show on those taken at higher temperature. λλ 3255.81, 3270.08, 3273.79 are strong and easily reversible lines. They maintain about the same relative strength except at low temperature, when the persistence of the first of the group justifies placing it in Class I. In this region, and throughout the spectrum, many lines remain about as strong at 2300° as at 2600°. Plates taken at 2100° showed lines of this type just appearing. Their absence in most cases at 2000° causes them to be placed in Class III.

Several strong lines occur in the violet, from λ 3908 to λ 4083. These are strong in the arc and reverse in the furnace at high temperature, retaining a considerable intensity at lower temperatures. From λ 4100 to λ 4700, the furnace gives no strong lines, and such lines as appear do not show at low temperature. The strong lines given in this region by the arc are uniformly classed as enhanced lines. Near λ 4750 a prominent group occurs, including the remarkable Class I lines λλ 4753.34 and 4779.59. These are not notably strong at high temperature, though their low density is caused partly by the beginning of reversal. At low temperature, however, they are much the strongest lines of the region, indeed, of the whole visible spectrum, with the exception of the very strong lines in the red.

Through the green and yellow the prominent lines are mostly arc lines which diminish gradually in strength with decreasing furnace temperature.

The region from λ 6146 to λ 6448 contains some very noteworthy lines. The stronger ones are relatively stronger in the furnace than in the arc. Two of them, λλ 6240 and 6306, have faint companions in the arc which have not been previously measured. These weak companions attain high intensities in the furnace and were measured on low-temperature furnace plates as λ 6239.64 and λ 6306.26. The lines in this red group are of three types, Class II and two varieties of Class I lines, some of the latter having about equal strength at all temperatures, and others being very strong at low temperature. A contrast between these two kinds of Class I lines is offered in the close pair λ 6305.94 and λ 6306.26, the former becoming three times as strong at low temperature as at high, while the latter retains the same strength at all the three temperatures. Especially notable is λ 6448.42. This has been measured in the arc only by Fowler. I have found it as a very faint line on a strong arc spectrum. In the furnace it is strong at all temperatures, but shows the behavior usually associated with strong arc lines, falling off rapidly with decreasing temperature. Fowler does not include it among the lines strong in the flame of the arc. It is difficult to account for a line of this type on the basis of dependence on intensity of excitation, as the arc conditions are distinctly unfavorable for it and it is relatively weak also at the low furnace temperatures. There being no reason to ascribe the line to an impurity, it must for the present be regarded as especially sensitive to some feature peculiar to the furnace radiation. λ 6345.06 shows a similar, though less pronounced, behavior in arc and furnace.

In addition to this type we have all gradations of intensity at different furnace temperatures for lines which are faint in the arc. Those of Class I A may be regarded as responding especially to low temperature. The many lines belonging to Class IV A, which, while faint in the arc, are strong at high furnace temperature, appear to respond only to a limited range of excitation.

2. *Enhanced lines.*—In the scandium spectrum we find the furnace able to produce with considerable intensity a number of enhanced lines. Of those given by Fowler in the visible region, and sufficiently strong in the arc to be entered in my list, 10 are in Class V, 8 in Class IV, and 7 in Class III. This means that at 2250°

some enhanced lines are given distinctly by the furnace. In most cases these are much stronger at 2600°, though λ 4247 is an exceptional line showing about the same strength at both temperatures. In every case the relative intensity in the arc as compared with the furnace is much greater than that of the regular arc lines.

In the furnace data given here, and previously, we have a confirmation of the view regarding enhanced lines expressed by Fowler in his paper on the scandium spectrum.[1] He called attention to the fact that in the spectra of titanium, calcium, strontium, and barium, as well as of scandium, the arc gives strong lines which have all the characteristics of enhanced lines. The furnace results show the relation among the enhanced lines of these several elements. Those of titanium[2] are just visible in the high-temperature furnace and are placed in Class V. Some of the scandium enhanced lines appear at medium temperature and go into Class III, while the H and K lines of calcium and the homologous pairs of strontium and barium[3] persist with considerable strength at temperatures as low as 1650°, and on account of their rapid increase at higher temperatures and in the arc are assigned to Class II.

Enhanced lines of different elements thus differ greatly as to the excitation required for their initial appearance. When given at all by the furnace, they agree in their rapid strengthening with increase of temperature and in the arc and finally in the great intensification shown in the spark. The last feature distinguishes them from the large group of high-temperature lines which in laboratory sources reach their maximum intensity in the arc and are unaffected or even weakened in the spark.

3. *Comparison of arc-flame and low-temperature furnace lines.*—By proper manipulation of the arc, a spectrum from the outer vapors can often be obtained which differs materially from that of the vapor in the direct path between the electrodes. The lines relatively strong in this "arc flame" are usually considered as the low-temperature lines, though in the arc the action of temperature

[1] *Op. cit.*, p. 52.

[2] *Mt. Wilson Contr.*, No. 76; *Astrophysical Journal*, **39**, 139, 1914.

[3] *Mt. Wilson Contr.*, No. 150; *Astrophysical Journal*, **48**, 13, 1918.

may be complicated by the vigorous oxidation taking place in the enveloping vapor.

Fowler[1] lists 29 of the principal arc-flame lines of scandium for the visible region and compares them with the corresponding sun-spot lines. All of these are strong in the furnace, usually at low temperature, though some belong in Class III. These lines, however, are strong also in the regular spectrum of the arc, in which the radiation of flame and core are superposed. In the same region the furnace gives 13 strong furnace lines not in Fowler's list. These are often stronger at low temperature than some of the arc-flame lines. All of them, however, are relatively weak in the arc, being lines with "A" after the class-number. This comparison shows how far the arc flame may be used to select low-temperature lines and also its limitation in this respect, in that it does not bring out with any notable intensity those lines which are faint in the core of the arc and, for some reason at present obscure, are strong in the furnace spectrum.

In the iron spectrum, the degree of extension into the flame of the arc of the various classes of furnace lines was studied by the write [2] and it was then remarked that lines of Class I A appear to be radiated almost entirely by the outer vapors but are inherently very faint. For the study of such lines the furnace conditions are very advantageous.

The fact that the low-temperature furnace in a partial vacuum gives all of the lines prominent in the scandium arc flame and a number besides, is evidence that the oxidation in the arc flame is not influential in producing the lines observed, but that they are due to the reduced temperature in this region of the arc.

4. *Band spectrum.*—The flutings prominent in the scandium arc burning in air were usually entirely absent in the furnace photographs. Only on some long exposures at low temperature was the head of the orange band at λ 6036 visible, and but one of these showed any development of structure. This evidence, as far as it goes, favors the view that the bands are due to the oxide, since, although the material was initially in the oxide form, it

[1] *Op. cit.*, p. 69.

[2] *Mt. Wilson Contr.*, No. 66; *Astrophysical Journal*, **37**, 239, 1913.

appeared to be reduced at once with the formation of a carbide, in which case the oxygen would unite with the carbon of the tube. The prolonged exposures in some cases may have allowed a little oxygen from the residue of air in the chamber to combine sufficiently with the scandium vapor to show the one band observed. If the flutings were due to scandium itself they should have appeared at one of the temperatures used, since in general the vacuum furnace is very effective in producing a banded spectum when this is due to the metal being vaporized.

5. *Comparison with solar and sun-spot spectra.*—The data now available furnish an extension of the observations of Fowler in regard to the scandium lines occurring in solar and spot spectra. In the first place a comparison of the lines in Table I with Rowland's "Table of Solar Spectrum Wave-Lengths" shows that, though only a few are identified by Rowland as belonging to scandium, and these in almost all cases enhanced lines, a large proportion of the scandium wave-lengths agree closely with those of solar lines. A number of these probable coincidences are listed by Lockyer and Baxendall.[1] The strength of scandium lines in the sun bears a rather close relation to the degree of electrical excitation in the laboratory, the strongest, graded 3, 4, or 5 on the Rowland scale, being enhanced lines or those strong in both arc and spark. Those showing no marked response to electrical excitation, but often strong in both furnace and arc, are faint in the sun, usually below intensity 1; while lines relatively strong in the furnace, designated by "A" after the class-number, are uniformly either of intensity 000 or 0000 in the sun, or entirely absent from Rowland's table. In this latter class the pronounced low-temperature lines λλ 4753 and 4780 have no counterparts in the solar spectrum and such powerful furnace lines as λλ 6194, 6259, 6306, 6379, 6413 are either lacking or just at the limit of visibility.

A comparison with the sun-spot spectrum showed a decided prominence of the lines which are strong in the furnace. For this purpose the lines in Table I of intensity 10 or higher in the furnace spectrum were selected. While some of them are indistinct or disturbed by blends, there is little question that these lines are

[1] *Proceedings of the Royal Society*, **74**, 538, 1905.

very generally present in the spot spectrum, and a striking increase appears regularly over the intensities in the solar disk.

The material available was used in two ways. First, a comparison was made with a list of spot lines compiled several years ago from spectrograms of moderate dispersion. In these the effect of the magnetic field in the spot was usually simply to widen the line, so that intensity estimates were more easily made than when the lines are resolved into magnetic doublets. In this list low-temperature scandium lines not visible in the solar spectrum often appeared in the spot with a value of at least 00 on the Rowland scale, while λ 6413.57 of this type reaches an intensity of 2 in the spot. Lines with a solar strength of 0000 or 00 usually strengthen to 2 or higher in the spot.

The second comparison was with the more recent large-scale spectra in which, by means of a nicol prism and compound quarter-wave plate, the intensity of the line and also its response to the magnetic field is recorded across the spot and over a portion of the disk on each side. In these spectrograms the relatively high intensity in the spot of the scandium furnace lines is very pronounced, and many fine cases of magnetic separation occur. Lines faint or invisible in the solar spectrum appear with considerable strength in the spot, even when resolved into two components. Some of these magnetic doublets are very wide, notable among them being the strong furnace lines λλ 5717, 6194, 6240, 6259, 6276, 6306.26, 6345, 6414. As is usual with magnetic resolutions, the average width decreases as we proceed toward shorter wave-lengths, though low-temperature lines such as λλ 4753 and 4780 are notable both for strength in the spot and for sensitiveness to the magnetic field. Judging from the spot spectrum, since there are no laboratory observations of the Zeeman effect for scandium, a uniformly strong response to the magnetic field characterizes this spectrum.

CONCLUSION

The varying response of spectrum lines to temperature change, observed in the study at different temperatures of the furnace spectrum of scandium, has concerned itself especially with the conditions of appearance and development of the three main types—

enhanced, arc, and low-temperature lines—which make up the spectrum. The scandium enhanced lines, as regards degree of excitation required, stand between the enhanced lines of titanium and those of the type of H and K of calcium. For the lines appearing in the arc spectrum the regular temperature classification is carried out. The group of arc lines given by the arc flame consists of prominent low-temperature lines, but, since many lines strong in the furnace are relatively weak in all parts of the arc, additional data on this type of line are supplied by the furnace spectrum.

The explanation advanced by Fowler as to the relative strength of the different types of scandium lines in the solar and sun-spot spectra is fully confirmed by their varying response to the furnace excitation. The strong low-temperature lines, faint or lacking in the disk, are distinctly brought out in the spot, and, especially in the red, show high sensitiveness to the magnetic field. A comparison of the Zeeman effect for scandium in the laboratory with that shown in spot spectra and a fuller listing of the scandium lines present in the sun are needed to give this element its due place among those on which solar studies are based.

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MAGNETIC ROTARY DISPERSION IN TRANSPARENT LIQUIDS

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ABSTRACT

Magnetic rotary dispersion of isotropic transparent media.—The electron theory as given by H. A. Lorentz is extended, and a formula for the rotation in a range of spectrum in which electrons of only a single type, with critical frequency c/λ_1, need be considered, is developed:

$$\theta=\pi(C_1h_1)\cdot Hl\cdot\frac{[1+\sigma(\mu^2-1)]^2}{\mu}\cdot\frac{\lambda_1^4\quad\lambda^2}{(\lambda^2-\lambda_1^2)^2}\,;\ \text{where } C_1h_1=\frac{Ne^4}{2\pi^2cm^2},$$

H is field strength, l is length, μ is refractive index, c is velocity of light, and σ is a constant which Lorentz puts equal to about $\frac{1}{3}$ and Voigt puts equal to zero. To test this theory, substances were chosen whose dispersion was known to conform to the theory of Lorentz, and the *magnetic rotations for carbon disulphide, α-monobromnaphthalene, benzene, nitrobenzene and ethyl iodide* for six wave-lengths from 436 to 620 $\mu\mu$ were determined with a cell 2 cm long in a field of 6480 gauss. The angles could be measured to $\frac{1}{10}°$ and were found to vary from 4° to 28°, increasing rapidly for each liquid with decreasing wave-length. Theoretical curves were computed, taking the values of C_1, λ_1 and μ from measurements by others. Below 590 $\mu\mu$ the experimental curves lie below the theoretical curves, the divergence increasing as the wave-length diminishes until the difference at 423 $\mu\mu$ is from 4 to 20 per cent. It is suggested that this discrepancy is due to the absorption of the violet end of the spectrum which was neglected in the theory. The results do not decide between the values 0 and $\frac{1}{3}$ for σ. The *values of e/m for the active electrons* may be computed from h_1 and vary from 0.5 to 1.78×10^7 e.m.u. according to the liquid and to the value of σ assumed.

1. *Introductory.*—In 1845 Faraday discovered that isotropic substances when placed in a strong longitudinal magnetic field rotate the plane of polarization of plane-polarized light. The obvious explanation of this, confirmed later by Brace, was the same as that offered by Fresnel for the rotation observed in certain crystals, viz., to consider the plane-polarized beam to be composed of two circularly polarized components which travel through the medium with different velocities, one greater and the other less than the velocity of the beam in the medium when unmagnetized. The variation of the angle of magnetic rotation with the wave-length of the light has been considered theoretically by several writers, and the theoretical relations have been applied to the experimental data available. Such data appear, however, to be rather meager, and it seemed desirable to pursue the subject afresh both theoretically and experimentally.

We do not presume to attempt a historical summary of the researches on the many aspects of this subject. For a summary of the earlier work we make reference to an article by P. Joubin.[1] Among the more important theoretical formulae for the magnetic-rotation angle in terms of the wave-length, magnetic field, etc. were those obtained by Joubin, Voigt,[2] and Drude.[3] For a summary of the experimental measurements of the magnetic-rotation angles we turn to the Landoldt-Börnstein *Physical Tables*. There the data have been tabulated in many cases as the *Verdet Constant*, which is defined to be the angle of rotation of plane-polarized light of specified wave-length produced by a medium 1 cm in length when magnetized by a longitudinal magnetic field of strength 1 gauss. These measurements were applied by Joubin and Drude to a dispersion theory of isotropic media, although for such a purpose the experimental data were hardly sufficiently numerous. With the exception of a few transparent liquids and solids the Verdet Constant was found either for unresolved light or for a single wave-length of monochromatic light. In the cases of those substances whose Verdet constant has been determined for a number of wave-lengths throughout the spectrum, one could introduce the values into a suitable dispersion formula to test a dispersion theory. Unfortunately, however, the refractive indices of most of these substances do not conform to any of the simpler dispersion formulae whose constants have a physical interpretation, and so these data are, from this standpoint, uninteresting.

In the present work transparent liquids were chosen whose dispersion conformed approximately to the electronic dispersion theory of H. A. Lorentz.[4] The angles of magnetic rotation of plane-polarized light for a series of wave-lengths of light in the visible spectrum were measured. A formula of Lorentz giving the relation between the refractive index and the strength of the magnetic field was modified to express the magnetic-rotation angle in terms of the wave-length, the field strength, and other quantities. The formula was found to agree approximately with experiment in

[1] *Thèses*, Faculté des Sciences, Paris, 1888.

[2] *Magneto- und Elektrooptik*, 1908.

[3] *Lehrbuch der Optik*, 1906.

[4] *Theory of Electrons*, 1909.

those regions of the spectrum wherein the assumptions upon which it was based were valid.

2. *Theoretical.*—Suppose plane-polarized light of wave-length λ *in vacuo* traverses a length l of a medium of refractive index μ for the wave-length in question with a velocity v. If the medium is placed in a magnetic field of strength H, such that the lines of magnetic force are parallel to the direction of propagation of the light, the medium becomes doubly refracting. The two circularly polarized components, which compose the plane-polarized beam, now pass through the medium with velocities v_1 and v_2, and the medium has the corresponding refractive indices μ_1 and μ_2, respectively.

Eddy, Morley, and Miller,[1] followed by Mills,[2] have shown by an interferometer method that

$$\frac{v_1+v_2}{2}=v, \tag{1}$$

or

$$\frac{1}{\mu_1}+\frac{1}{\mu_2}=\frac{2}{\mu}. \tag{2}$$

This was determined by measurements on carbon disulphide. The accuracy of the work was none too great, because the experiment was a difficult one; but, within the error of observation, the foregoing relation was true. It may be noticed that from general considerations one would not expect (1) to be exactly true, and further that one would expect the difference between (1) and the exact truth to be small. However, we now assume that (1) is true for all the substances and throughout the wave-length range used in this investigation.

Let θ be the observed angle in radians of the rotation of the plane of polarization produced by the magnetized medium. It is easily shown[3] that

$$\mu_2-\mu_1=\frac{\lambda\theta}{\pi l}, \tag{3}$$

where λ is the wave-length of the light *in vacuo*.

[1] *Physical Review*, **7**, 283, 1898.

[2] *Ibid.*, **18**, 65, 1904.

[3] Drude, *loc. cit.*, p. 396.

Solving (2) and (3) for μ_1 and μ_2, and considering $\frac{\lambda\theta}{\pi l}$ small compared with μ, we obtain

$$\left.\begin{aligned} \mu_1 &= \mu - \frac{\lambda\theta}{2\pi l}, \\ \mu_2 &= \mu + \frac{\lambda\theta}{2\pi l}. \end{aligned}\right\} \tag{4}$$

Let us turn to a consideration of the dispersion theory in this connection. We restrict the discussion to isotropic, transparent media in which the temperature remains constant. We use the electron theory of dispersion as given by H. A. Lorentz (*loc. cit.*). Let ξ and E_x be the X components of the displacement of the electron from its equilibrium position and the electric force, respectively. η, ζ, E_y and E_z are the Y and Z components of these quantities; they are all expressed in c.g.s. electromagnetic units. The components of the "restoring force" with which the medium acts upon the electron are $f\xi$, $f\eta$, and $f\zeta$. The charge on the electron in c.g.s. electromagnetic units is e, its mass is m. N is the number of such electrons per unit volume. σ is a constant which Lorentz has shown to be approximately one-third for isotropic media. The external magnetic field is denoted by H in c.g.s. electromagnetic units. We shall suppose H to have the direction of the axis of Z, which is also the direction of the propagation of the light. The magnetic permeability of the medium is taken as unity.

We find for the equations of motion of the dispersion electron of a single type

$$\left.\begin{aligned} m\frac{d^2\xi}{dt^2} &= e(E_x + 4\pi c^2\sigma N e\xi) - f\xi + eH\frac{d\eta}{dt}, \\ m\frac{d^2\eta}{dt^2} &= e(E_y + 4\pi c^2\sigma N e\eta) - f\eta - eH\frac{d\xi}{dt}, \\ m\frac{d^2\zeta}{dt^2} &= e(E_z + 4\pi c^2\sigma N e\zeta) - f\zeta. \end{aligned}\right\} \tag{5}$$

Let ϵ be the base of natural logarithms, and let all dependent variables of (5) contain the time only in the factor $\epsilon^{i\frac{2\pi c}{\lambda}t}$ where $\frac{c}{\lambda}$

is the frequency, λ the wave-length of the vibration *in vacuo*, and c the velocity of light *in vacuo*. The solution of (5) gives the refractive index μ as determined by the relation

$$\frac{1}{\sigma+\frac{1}{\mu^2-1}}=\frac{C_s}{\frac{1}{\lambda_s^2}-\frac{1}{\lambda^2}\pm\frac{H}{\lambda}h_s}, \tag{6}$$

where

$$C_s=\frac{N_s e_s^2}{\pi m_s},$$

$$h_s=\frac{e_s}{2\pi c m_s}.$$

The subscript s is used to denote the sth type of electron. $\frac{c}{\lambda_s}$ is the frequency of the natural undamped vibration of this electron. When the plus sign in equation (6) is used, μ is the μ_2 of (4), and when the minus sign is used μ is the μ_1 of (4).

Equation (6) describes μ in terms of the constants of a single type of dispersion electron. There may be other types of dispersion electrons in the medium with constants peculiar to the type, so that in the more general case the right-hand member of (6) becomes a summation of similar terms, one term for each type. For this case the complete dispersion formula is

$$\frac{1}{\sigma+\frac{1}{\mu^2-1}}=\sum\frac{C_s}{\frac{1}{\lambda_s^2}-\frac{1}{\lambda^2}\pm\frac{H}{\lambda}h_s}. \tag{7}$$

We assume we are dealing with a region of the spectrum in which the change of the refractive index with wave-length is determined by the electrons of a single type, so that in the summation of (7) all the terms except one may be replaced by a quantity q_1 which is independent of λ and H. Then (7) becomes

$$\frac{1}{\sigma+\frac{1}{\mu^2-1}}=q_1+\frac{C_1}{\frac{1}{\lambda_1^2}-\frac{1}{\lambda^2}\pm\frac{H}{\lambda}h_1}, \tag{8}$$

where

$$\left.\begin{aligned} C_1&=\frac{Ne^2}{\pi m}, \\ h_1&=\frac{e}{2\pi cm}. \end{aligned}\right\} \tag{9}$$

A special case occurs when H is zero. For this (8) becomes equivalent to the well-known dispersion formula of Lorentz:

$$\frac{1}{\sigma+\frac{1}{\mu^2-1}}=q_1+\frac{C_1}{\frac{1}{\lambda_1^2}-\frac{1}{\lambda^2}}. \tag{10}$$

Introducing (4) into (8) gives:

$$\frac{1}{\sigma+\frac{1}{\left(\mu\pm\frac{\lambda\theta}{2\pi l}\right)^2-1}}=q_1+\frac{C_1}{\frac{1}{\lambda_1^2}-\frac{1}{\lambda^2}\pm\frac{H}{\lambda}h_1}. \tag{11}$$

It was expedient to transform this relation to one more amenable to arithmetical computation. This was readily done because $\frac{\lambda\theta}{2\pi l}$ and $\frac{H}{\lambda}h_1$ were smaller than the other quantities appearing with them in the denominators by a different order of magnitude.

Equation (11) becomes to a close approximation

$$\theta=\pi C_1 h_1 Hl\frac{[\sigma(\mu^2-1)+1]^2}{\mu}\frac{\lambda_1^4\lambda^2}{(\lambda^2-\lambda_1^2)^2}. \tag{12}$$

θ is in radians.

We notice that if we use either the two plus signs of (11) or the two minus signs, we arrive at the same equation for (12). This shows that equation (8) is in close agreement with (4). We notice also that, other quantities remaining constant, θ is proportional to l and H in turn. This agrees with the experimental measurements of Rodger and Watson[1] and Dubois.[2]

[1] *Phil. Trans. Roy. Soc.*, A **186**, 621, 1896. This paper contains further references.

[2] *Wied. Ann.*, **35**, 137, 1888.

If σ is given the value $\frac{1}{3}$, equation (12) becomes

$$\theta=\pi C_1 h_1 H l \frac{(\mu^2+2)^2}{9\mu} \frac{\lambda_1^4 \lambda^2}{(\lambda^2-\lambda_1^2)^2}. \tag{13}$$

When σ is placed equal to 0, equation (12) becomes

$$\theta=\frac{\pi C_1 h_1 H l}{\mu} \frac{\lambda_1^4 \lambda^2}{(\lambda^2-\lambda_1^2)^2}. \tag{14}$$

Formula (14) agrees with one given by Voigt[1] when his quantities are expressed in c.g.s. electromagnetic units.

To compare the experimental measurements with the theory the following procedure was used. Values of the refractive index of the liquid under investigation for wave-lengths in the visible spectrum were taken from data published by others. Three values of μ and the corresponding three values of λ were substituted in equation (10), and the three constants q_1, C_1, and λ_1 were computed. Using these three constants, a fourth value of λ was substituted in (10) and the value of μ calculated. If this value of μ agreed approximately with the observed value, the substance was considered to conform to the Lorentz dispersion equation (10) for the region of the spectrum in question. The values of θ for a number of wave-lengths in the visible spectrum were measured for a known length l of the liquid subjected to a known magnetic field H. The constant h_1 was then determined by substituting in equation (13) the values of H, l, C_1, λ_1, and the values of λ, θ, and μ for a specified wave-length. θ was then computed by means of (13) for the other wave-lengths, and the computed value was compared with the observed value.

3. *Experimental arrangements.*—The arrangement of the apparatus is shown in plan in Figure 1. A gas-filled lamp with a spiral tungsten filament and a mercury-vapor lamp served as sources of light. The mercury-vapor lamp was used for observations at wave-lengths 435.9 $\mu\mu$ and 546.1 $\mu\mu$; the tungsten lamp was used for observations from the green to the red end of the spectrum.

[1] *Loc. cit.*, p. 130.

The source of light, shown by A, Figure 1, was focused on the slit s_1 of the spectrograph by a lens l_1, 3 cm in diameter and of focal length 22 cm.

The spectrograph consisted of the Littrow mounting of a plane grating. The grating had a ruled area 6 cm by 7.5 cm and was ruled 15,000 lines to the inch. The cone of light from slit s_1 was reflected by a right-angle glass prism through the large lens l_2, 10 cm in diameter and with a focal length of 75 cm. The spectrum was brought to a focus at slit s_2. The grating was mounted on a turntable which could be rotated from the outside of the case containing the spectrograph, so that various wave-lengths of light could be made to pass through the second slit. The grating possessed a bright first order, and this first-order spectrum was

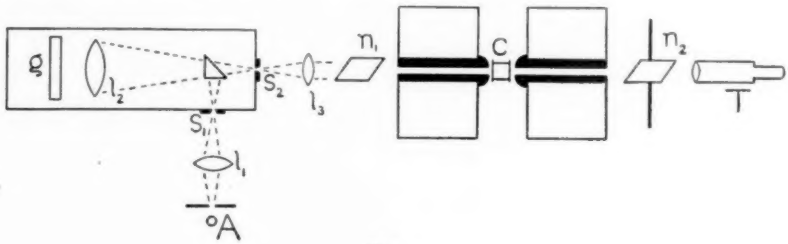


FIG. 1

used throughout the present work. The dispersion was such that with slit s_2 0.25 mm wide a beam of light containing a wave-length range of 5 A, or 0.5 $\mu\mu$, passed through. Both slit s_1 and slit s_2 were always 0.25 mm in width.

The monochromatic beam of light emerging from slit s_2, after being rendered parallel by a lens l_3, 2.5 cm in diameter and of focal length 12.5 cm, passed through the polarizing nicol n_1 and through the pierced pole pieces of a Ruhmkorff magnet between which the cell C containing the liquid under investigation was placed. The light then traversed the analyzing nicol prism n_2 and finally entered the telescope T. The lenses l_1, l_2, and l_3 were achromatic doublets.

The cell which contained the liquid was made of glass. A short length, about 2 cm, of glass tubing of internal diameter 1 cm, to which had been sealed a small side tube, was ground until the

ends were plane-parallel to within 0.003 cm. Glass plates with optically plane surfaces were cemented to this. Rubber cement, a mixture of lead oxide and glycerine, and LePage's liquid glue, were found useful as cements in various cases. The cell was filled through the side tube. The length of the column of liquid was found by subtracting the thickness of the end plates from the external length of the cell. The liquids used were obtained from the Chemical Laboratory and were of a high degree of purity.

The pole pieces of the Ruhmkorff magnet were elliptical in shape to give a uniform magnetic field; they were adjusted to be 2.5 cm apart. To calibrate the magnet, the angle of rotation θ for sodium light was observed for a known length of carbon disulphide for a current of 26.0 amperes through the magnet. From θ and from the value of the Verdet constant of carbon disulphide for sodium light, which has been carefully determined by Rodger and Watson (*loc. cit.*), the average strength of the magnetic field between the pole pieces was calculated, and was found to be 6480 gauss. This field strength was used for all the rotation measurements of this paper. It was found by tests that the rotation angle θ had the same absolute value for the magnetic field direct and reversed. This indicated that hysteresis effects in the magnet were negligible.

The mounting of the analyzing nicol carried a circular scale which measured the rotation angles to one-tenth of a degree of arc. It was found that the analyzer could be set for extinction for all the wave-lengths with a precision of two-tenths of a degree. The values of the rotation angle were in all cases the means of at least four measurements, two taken with the field direct and two with the field reversed. It was considered that the mean angle was correct to one-tenth of a degree of arc.

4. *Errors and corrections.*—No correction was made for the error due to scattered light. There were two ways in which scattered light might introduce systematic error into the rotation measurement, the first arising from light scattered by the grating, and the second from multiple reflections by the surfaces at the ends of the cell which held the liquid. To determine the effect of the light scattered by the grating, the beam issuing from slit s_2 was examined with a transmission grating. When the tungsten lamp

was used as the source of light, it was found that the beam contained light of foreign wave-lengths of rather feeble intensity; in the case of the mercury lamp this foreign light appeared still weaker. Upon looking into the telescope and setting the analyzer for extinction it was found that the image of slit s_1 did not fade out against an absolutely black background. Very faint light was seen in the field, due no doubt to the light scattered by the grating and to imperfections throughout the optical system. The settings for extinction could, however, be made with precision, and it was deemed that the extraneous light introduced no appreciable systematic errors. Any error due to multiple reflections from the surfaces of the ends of the glass cell was avoided by rotating the cell slightly until these surfaces were not perpendicular to the beam of light. The error which this caused in the determination of l was negligible.

TABLE I

λ	θ
436 $\mu\mu$	1.7°
503	1.3
546	0.9
579	0.85
589	0.85
620	0.8

It had been feared that errors due to temperature would be troublesome, but it was found that these fears were needless. To carry out a complete series of measurements of the rotation angle for five wave-lengths in the visible spectrum required about an hour. During this time the magnet heated up, and the temperature of the liquid in the cell increased. The increase was never more than 3° C. In the case of carbon disulphide the temperature coefficient of the Verdet constant for sodium light has been determined.[1] A three-degree change in temperature changed the Verdet constant by about 0.5 per cent. In the present case it was considered that errors due to temperature changes were for the most part less than 0.5 per cent, and that it was unnecessary to arrange a more accurate control of the temperature of the liquid in the cell.

[1] Rodger and Watson, *loc. cit.*

The rotation angle θ due to the liquid alone was obtained from the observed rotation angle produced by the liquid in the cell by subtracting from this observed angle the angle of rotation produced by the empty cell for the wave-length in question. Table I shows θ for the two glass plates on the ends of the cell. The thickness of the two plates together was 0.315 cm, the field strength was 6480 gauss. The magnetic-rotation angles plotted in the figure have in all cases been corrected for the effect of the glass ends of the cell.

5. *Carbon disulphide.*—The data for this substance and the results of the calculations are given in Table II and Figure 2. We shall discuss these in some detail, and shall avoid a repetition of the discussion for the other substances. The observed values of θ, shown by circles in Figure 2, have been plotted as ordinates against wave-lengths as abscissae; a smooth line, curve 1, has been drawn through them. The strength of the magnetic field, the length of the layer of liquid, and the temperature at the beginning and the end of the experiment are shown in the first two lines of Table II.

Verdet[1] has recorded relative values of θ for carbon disulphide for a number of wave-lengths in the visible spectrum. The magnetic field used was not mentioned. For the sake of comparison the values given by Verdet have been reduced to agree with curve 1 for λ 589.3 $\mu\mu$ and are shown by crosses in Figure 2. Joubin (*loc. cit.*) also carried out measurements on carbon disulphide. Neither the magnetic field, nor the length of the liquid, nor the temperature were recorded. By a coincidence his value for θ at λ 589.3 $\mu\mu$ was the same as that of curve 1, namely 10°3. His values have been plotted as dots in Figure 2. We think that the measurements of curve 1 were correct, for they were repeated with precision a few weeks later.

In order to introduce these experimental results into the dispersion formula (10) we assume that the absorption of carbon disulphide is inappreciable for the visible wave-lengths in question. Such an assumption is manifestly not accurate, because this substance absorbs the blue end of the spectrum to a certain extent.

[1] Verdet, *Oeuvres complètes.*

We take the values of the refractive index found at 20°0 C. by Flatow. These and the corresponding wave-lengths are shown in the third and fourth lines of Table II. Substituting these values into formula (10), the values of the constants C_1, q_1, λ_1 were computed and are tabulated in the fifth line of Table II. The agreement between (10) and observations was tested by using the foregoing values of the constants and computing μ for λ 394 $\mu\mu$ to be 1.7043. The observed value was 1.70226, and

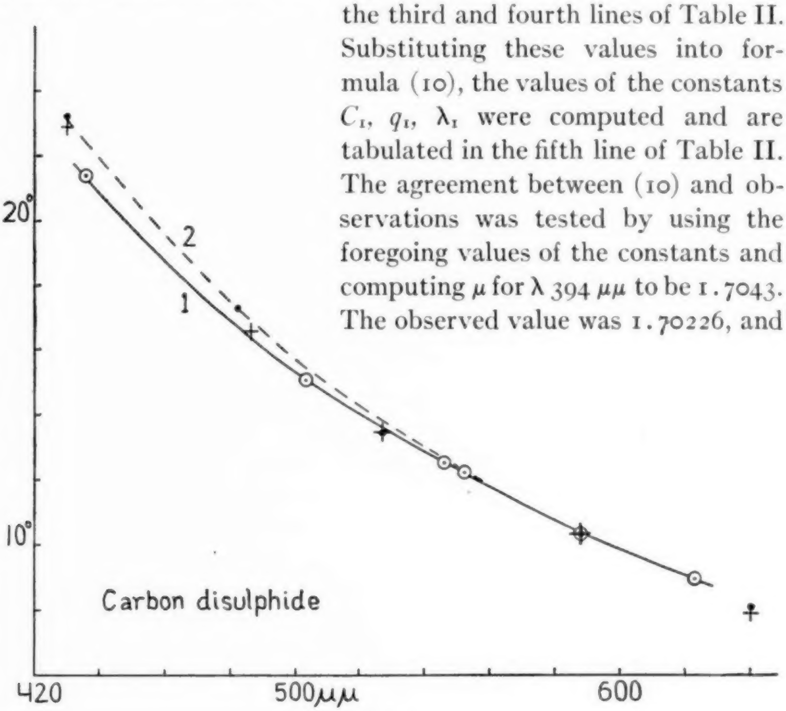


FIG. 2

TABLE II

CARBON DISULPHIDE

H=6480 gauss	Temp. 21°2 to 22°0 C.		l=2.272 cm
λ 441.6 $\mu\mu$	508.6	589.3	
μ 1.67180	1.64586	1.62806	at 20°0 C.
Observed by Flatow, *Ann. d. Phys.*, **12**, 85, 1903.			
C_1=10.381×10^8	q_1=0.57272	λ_1=204.2 $\mu\mu$	
λ 394 $\mu\mu$	observed μ=1.70226		
	calculated μ=1.7043		
h_1=3.95×10^{-5} for λ 589.3 $\mu\mu$			

the agreement was considered sufficiently close. These numbers are given in lines 7 and 8 of Table II. The dispersion of carbon disulphide and α-monobromnapthalene has been more fully discussed in a former paper.[1]

The constant h_1 was then determined by substituting in equation (12) the values of H, l, C_1, and λ_1 given in Table II, and the values of λ, θ, and μ for the wave-length 589.3 $\mu\mu$. This gave 3.95×10^{-5} for h_1, as shown in the last line of Table II, when all the quantities were expressed in c.g.s. and c.g.s. electromagnetic units.

The constants of (13) were now completely known, and (13) was then used to compute the values of θ for the range of the spectrum under investigation. The computed curve is shown by the dotted line, curve 2, of Figure 2. It is seen that the agreement between the observed and calculated values of θ was fairly close for the longer wave-lengths, but that for the shorter wave-lengths the theoretical value was greater than the observed value, the difference between the two values increasing as the wave-length decreases. This difference between theory and experiment may be attributed, in part at least, to the neglect of the effect of absorption in the theory. The discrepancy was in the right direction to be attributed to this effect, for the introduction into the theoretical formula of terms denoting absorption will produce a decrease in the computed values of θ for the shorter wave-lengths.

In his treatise on optics Drude[2] has derived two theoretical expressions for the magnetic rotatory dispersion of isotropic media, one based on the "molecular stream" hypothesis, and the other on the "Hall effect" hypothesis. When written in a form to show the connection between θ and λ, neglecting absorption, the two formulae were, respectively,

$$\theta=\frac{\mu}{\lambda^2}\left(a_1+\frac{a_2\lambda^2}{(\lambda^2-\lambda_1^2)}\right) \tag{15}$$

and

$$\theta=\frac{1}{\mu\lambda^2}\left\{a_3+\frac{a_4\lambda^4}{(\lambda^2-\lambda_1^2)^2}\right\} . \tag{16}$$

[1] *Astrophysical Journal*, **46**, 1, 1917.

[2] *Loc. cit.*, p. 406.

a_1, a_2, a_3, and a_4 were quantities which involved the masses, the charges, and the numbers of the "bound" and "free" electrons, the strength of the magnetic field, the length of the medium, etc. The refractive index wave-length relation in this connection was

$$\mu^2 = a_5 + \frac{a_6}{\lambda^2 - \lambda_1^2}. \tag{17}$$

Drude applied these equations to Verdet's magnetic-rotation measurements on carbon disulphide and creosote in the following manner. Using the value of λ_1 obtained from (17) and two known values each of θ and λ, the remaining two constants of (15) or (16) were determined. The θ-λ curve from either formula, which thus traversed two of the observed points, was found to pass closely to the remaining observed points. We do not believe, however, that the agreement found in this way between theory and experiment possesses great significance. One would not expect a theory which neglected absorption to portray the observations with great exactness.

Joubin (*loc. cit.*) has also derived a formula for the dispersion of magnetic rotation of somewhat the same type as (15). He applied this to the observations of rotations of carbon disulphide and creosote, which were measured for the purpose, in much the same manner as done by Drude.

6. *a-monobromnaphthalene.*—This substance was investigated in a manner similar to that described in the case of carbon disulphide. Table III shows a portion of the data and the results of the calculations. This table has been compiled exactly as was Table II for carbon disulphide, and therefore requires no further explanation. The observed values of θ have been plotted as circles in Figure 3, and a smooth line, curve 1, has been drawn through them. The computed values of θ from equation (13), using the constants of Table III, are shown by the dotted line, curve 2, of Figure 3. The differences between the observed and theoretical values are similar to those noted for carbon disulphide.

7. *Benzene.*—The values of θ for this substance were determined throughout the visible spectrum. These are shown by circles in Figure 4, through which a smooth line, curve 1, has been

passed. The Verdet constant of benzene was found by Jahn[1] to be 0.0297 minutes of arc for sodium light. From the data of Figure 4 and Table IV we obtain 0.0291 minutes of arc for the same wave-length. Jahn's value is not far different from this.

The computed values of θ from equation (13) are shown by the dotted line, curve 2, of Figure 4. It is seen that the differences between the observed and theoretical values are of the same character as those of the previous cases.

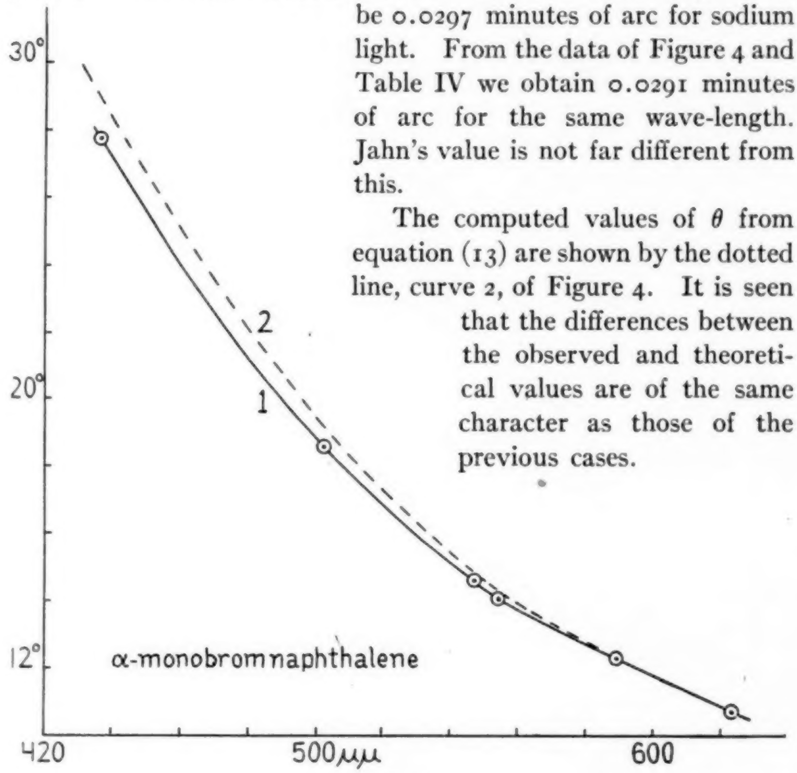


FIG. 3

TABLE III

α-MONOBROMNAPHTHALENE

$H=6480$ gauss	Temp. 23°.5 to 20°.7 C.		$l=2.272$ cm
λ 434 $\mu\mu$	486	589	
μ 1.70433	1.68245	1.65876	at 19°.4 C.
Observed by Brühl, *Ber. Chem. Ges.*, **22**, 388, 1897.			
$C_1=7.398\times10^8$	$q_1=0.70889$		$\lambda_1=215.6\ \mu\mu$
λ 656 $\mu\mu$	observed $\mu=1.64995$		
	calculated $\mu=1.6500$		
$h_1=5.02\times10^{-5}$ for λ 589.3 $\mu\mu$			

[1] *Wied. Ann.*, **43**, 280, 1891.

8. *Nitrobenzene.*—The results of the work on this substance are shown in Table V and Figure 5. The observed values of θ were plotted as circles in Figure 5, and a smooth line, curve 1, has been drawn through them. The computed values of θ from equation (13) are shown by the dotted line, curve 2.

Discrepancies of the same character exist between the observed and theoretical values of θ as were noticed in the preceding cases, but perhaps greater in magnitude.

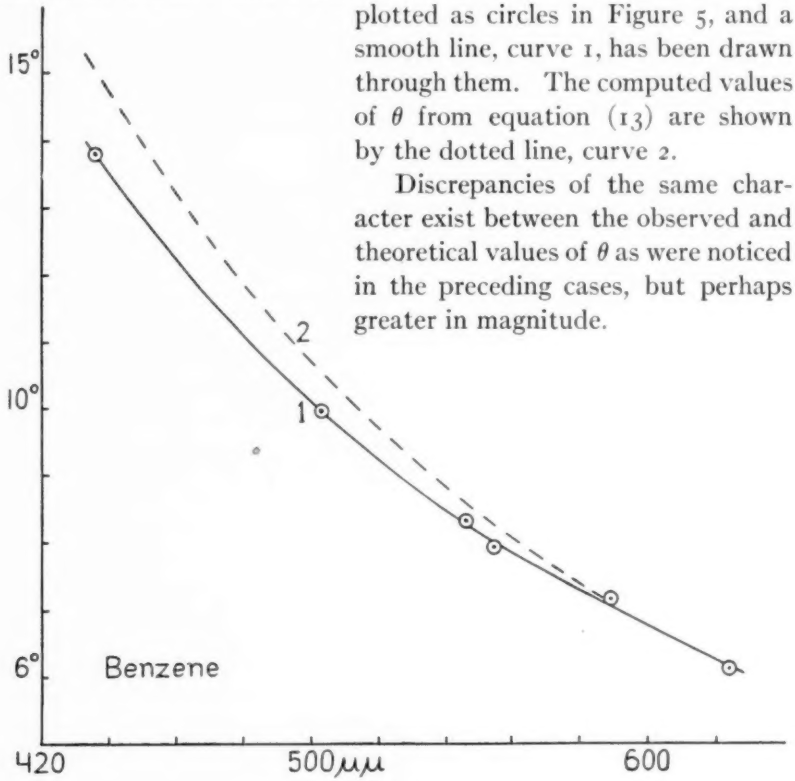


FIG. 4

TABLE IV

BENZENE

$H=6480$ gauss	Temp. 22°9 to 24°6 C.		$l=2.280$ cm
λ 434 $\mu\mu$	486	589	
μ 1.52380	1.51323	1.50111	at 20°0 C.
Landoldt-Börnstein Tables.			
$C_1=11.643\times 10^8$	$q_1=0.49898$	$\lambda_1=173.8\ \mu\mu$	
λ 656 $\mu\mu$	observed $\mu=1.49646$		
	calculated $\mu=1.4964$		
$h_1=5.34\times 10^{-5}$ for λ 589.3 $\mu\mu$			

9. *Ethyl iodide.*—The circles of Figure 6 show the observations on this substance; a smooth line, curve 1, has been drawn through the observed points. Other data are given in Table VI. Perkin[1] had determined the Verdet constant of ethyl iodide to be 0.0296 minutes of arc for sodium light. From the present data we find 0.0300 for this wavelength. The two values are not greatly at variance. The computed values of θ from equation (12) are shown by the dotted line, curve 2,

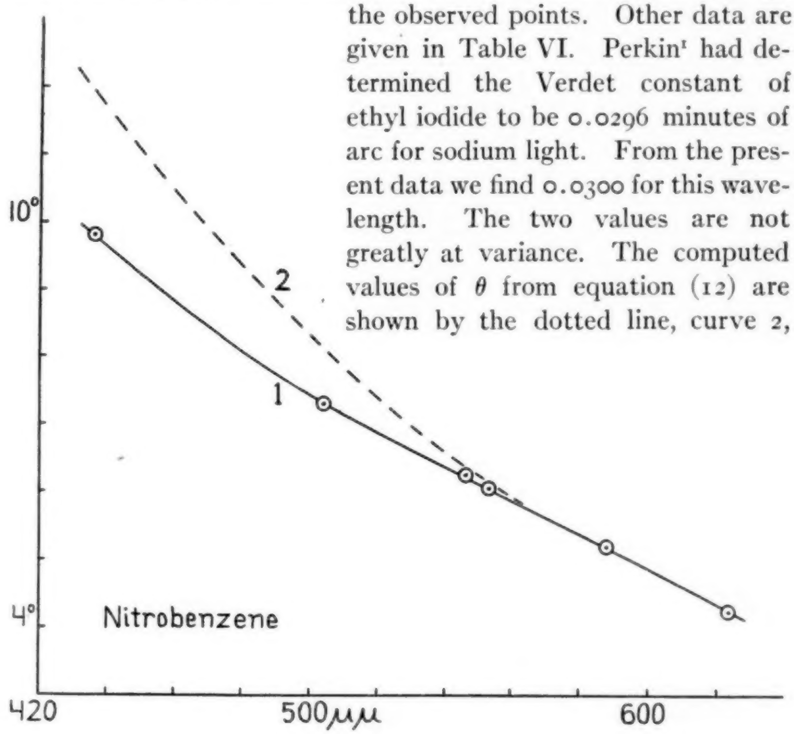


FIG. 5

TABLE V

NITROBENZENE

$H=6480$ gauss			$l=2.272$ cm
	Temp. 21°.4 to 22°.6 C.		
λ 486.2 $\mu\mu$	589.3	656.3	
μ 1.57165	1.55319	1.54641	at 20°.0 C.
Landoldt Börnstein Tables.			
$C_1=6.066\times10^8$	$q_1=0.62924$		$\lambda_1=217.13\ \mu\mu$
λ 434.1 $\mu\mu$	observed $\mu=1.5895$		
	calculated $\mu=1.5888$		
$h_1=2.56\times10^{-5}$ for λ 589.3 $\mu\mu$			

[1] Smithsonian Physical Tables, 1920.

Figure 6. It is seen that the discrepancies between the observed and theoretical values are similar to those of the preceding cases.

10. *Discussion of results.*—It has been demonstrated that the formula (13), which has been developed from the electron theory of Lorentz, served to express with a certain exactitude the dispersion of the magnetic rotation of certain liquids throughout the visible spectrum. For the longer

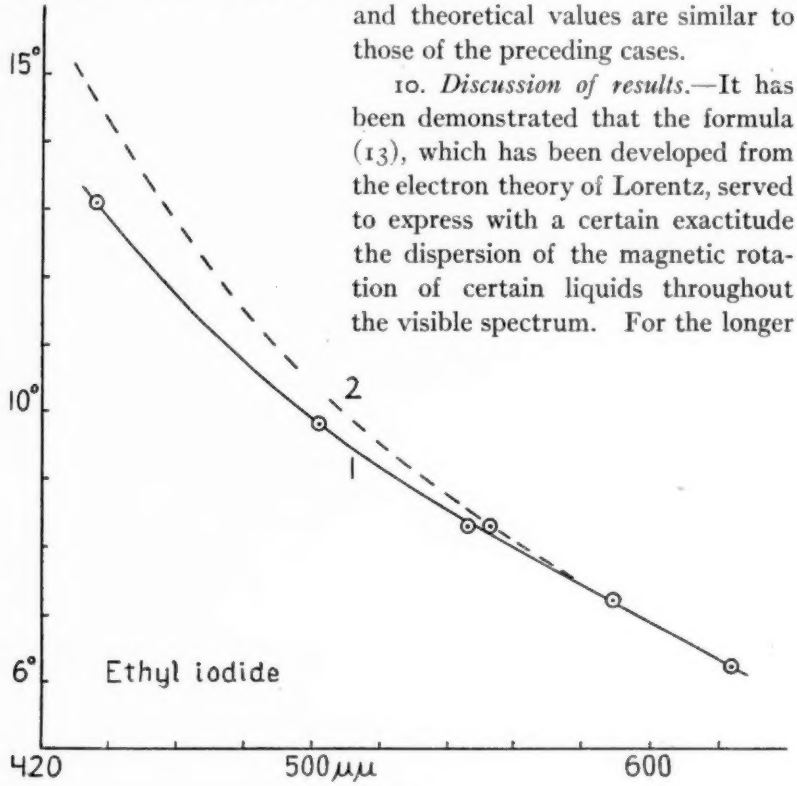


FIG. 6

TABLE VI

ETHYL IODIDE

H = 6480 gauss	Temp. 22°.7 to 24°.3 C.		l = 2.220 cm
λ 486.2 $\mu\mu$	589.3	656.3	
μ 1.52356	1.51203	1.50738	at 20°.0 C.
Observed by Lorentz, *Wied. Ann.*, **11**, 70, 1880.			
$C_1 = 12.782 \times 10^8$	$q_1 = 0.50762$	$\lambda_1 = 168.0\ \mu\mu$	
λ 434.1 $\mu\mu$	observed $\mu = 1.53437$*		
	calculated $\mu = 1.5336$		
$h_1 = 5.90 \times 10^{-5}$ for λ 589.3 $\mu\mu$			

* Haagen, *Pogg. Ann.*, **131**, 117, 1866.

wave-lengths of the visible spectrum the agreement between theory and experiment is quite close. For the shorter wave-lengths discrepancies occur which increase as the wave-length decreases. The liquids investigated above possess strong absorption in the ultra-violet and appreciable absorption in the blue end of the spectrum. We may therefore reasonably attribute the discrepancy between theory and experiment in large part to the neglect of absorption. If absorption is taken into account in our equations, λ_1 is increased, and in general the modification which the constants of the equations undergo is such as to bring the theoretical values of θ into closer agreement with the observed ones. The absorption of these substances for light has, however, not been measured accurately, and it seems unprofitable at this time to consider its effect numerically.

11. *The values of e/m.*—From the known value of h_1, the ratio of the charge to the mass of the electron may be calculated by

TABLE VII

	h_1	e/m
Carbon disulphide	3.95×10^{-5}	0.74×10^{7}
α-monobromnaphthalene	5.02	0.95
Benzene	5.34	1.01
Nitrobenzene	2.56	0.49
Ethyl iodide	5.90	1.12

means of formula (9). This has been done and the results are shown in Table VII; e/m is expressed in c.g.s. electromagnetic units.

Becquerel[1], Voigt,[2] and Siertsema[3] have derived formulae for the dispersion of the magnetic rotation, by means of which e/m can be found as soon as the Verdet constant and the dispersion $d\mu/d\lambda$ of a substance for the same wave-length λ are known. These three formulae reduce to the same one, namely

$$\frac{e}{m}=\frac{2c}{\lambda}\frac{\theta}{lH}\div\frac{d\mu}{d\lambda}. \tag{18}$$

[1] *Comptes rendus*, **125**, 679, 1899.

[2] *Wied. Ann.*, **67**, 351, 1899.

[3] *Comm. Lab. Leiden*, No. 82, 1902.

Using (18), Siertsema computed e/m for air, carbon dioxide, hydrogen, water, carbon disulphide, and quartz. The numbers varied from 0.75×10^7 to 1.77×10^7.

12. *The effect of σ upon the calculations.*—The value of the quantity σ has no very critical effect upon the variation of θ with wave-length. The calculated curves of the diagrams have been obtained by the use of $\frac{1}{3}$ for σ in formula (12). If σ is put equal to 0 in (12), we arrive at Voigt's formula (14), and if this is used to calculate the change of θ with λ we find values of θ which are a trifle less than those obtained from (13). They are about 1 per cent less at λ 434 $\mu\mu$, but are practically the same for wave-lengths greater than 500 $\mu\mu$. If σ is given values greater than $\frac{1}{3}$, the computed values of θ are found to be somewhat greater than the values given by (13), and therefore in greater discordance with the observed values. For example, in the case of carbon disulphide, if $\sigma=\frac{2}{3}$, θ is 25°1 at 434 $\mu\mu$.

The values of e/m change to some extent when σ is given different values. This is shown for carbon disulphide in Table VIII. We conclude that, as far as the present data are concerned,

TABLE VIII

σ	e/m
0	1.78×10^7
$\frac{1}{3}$	0.74
$\frac{2}{3}$	0.30
1	0.25

it makes little difference whether σ is 0 or $\frac{1}{3}$. If, however, σ is increased above $\frac{1}{3}$, the discrepancies between the theory and the observations become greater.

In conclusion the authors take pleasure in expressing their thanks to Dr. J. S. Ames for valuable and constructive criticism.

Johns Hopkins University
February 1921

A STUDY OF ARC-CATHODE SPECTRA

By ARTHUR ST. C. DUNSTAN and BENJAMIN A. WOOTEN

ABSTRACT

Arc spectra; relative intensity of metallic lines at anode and cathode.—A series of experiments were performed to *test various suggested explanations* for the fact, amply verified by the authors, that metallic lines are always stronger at the cathode when the metal is introduced symmetrically. A horizontal arc inclosed in a furnace was fed with metallic vapor (Sr, Ba, Li, Cu or Pb) supplied by an alternating current arc 5 cm below. Water-cooling either electrode had no effect, and attempts to obtain separation of the vapors electrolytically or by electrostatic action failed. The spectrum of a 60-cycle alternating current arc taken through a rotating sector synchronized to transmit light during only half of each cycle, was the same as that of a direct current arc. This proves that the phenomenon is fully developed in 1/120 second and makes it unlikely that it is due to the transference of the vapor from one electrode to the other either thermally or electrically. When small pellets of metallic salt were dropped through the arc, the spectrum lines were fish-shaped with the heads toward the cathode. The evidence as a whole indicates that the light is due chiefly to bombardment of the metallic vapor by electrons from the cathode.

Variations with atomic weight.—In general the ratio of cathode to anode intensity was found to decrease as the atomic weight increased.

The fact that in the spectra from the anode of an electric arc the carbon bands are strongest, while in those from the cathode the metallic lines are strongest, was first shown by Lockyer's discovery of the long and short lines.

The subject has been investigated by Thomas,[1] Miss Baldwin,[2] Foley,[3] Beckmann,[4] Humphreys,[5] Oellers,[6] and Konneman.[7]

In general the explanations offered may be divided into two classes, (1) thermal, (2) electrical. In the first class are included such processes as convection, distillation, differences of temperature, etc., all of which ultimately depend upon heat. In the second class may be grouped electrolysis, electronic action (as suggested by Humphreys), and possibly a direct action of the difference of potential between the electrodes of the ionized gases surrounding the arc. An attempt has been made to study the question under as definite, reproducible conditions as possible, and efforts have been made to vary one factor at a time.

[1] *Comptes rendus*, **119**, 728, 1894.

[2] *Physical Review*, **3**, 370 and 448, 1896. [3] *Ibid.*, **5**, 129, 1897.

[4] *Zeitschrift für wissenschaftliche Photographie*, **4**, 335, 1906.

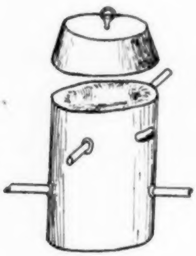
[5] *Astrophysical Journal*, **27**, 200, 1908.

[6] *Zeitschrift für wissenschaftliche Photographie*, **10**, 374–392, 1912.

[7] *Ibid.*, **12**, 65–76 and 123–124, 1913–1914.

In order to avoid the influence of convection currents in the air surrounding the arc, a horizontal arc inclosed in a refractory vessel was used, and in order to be independent of the fortuitous distribution of substances occurring as impurities in the carbons, elements were used which either did not occur as impurities, or occurred in the carbons in such small quantities that their spectral lines did not appear with the exposures given. It was found that Sr, Ba, Li, Cu, and Pb either did not occur in the carbons used, or occurred in very small quantities, and hence these elements were most often used, though for the sake of comparison others were occasionally employed. Strontium was the element generally used, in the form of sulphate.

The arc was surrounded by a nearly homogeneous atmosphere of the vapor of the metal, this vapor being supplied by another arc. The arrangement used is shown in the accompanying figure.



The lower carbons, forming what may be called the vaporizing arc, were bored out and packed with a salt of the metal studied. This arc vaporized the metal which diffused through the furnace. The temperature of the furnace was sufficiently high to prevent any considerable condensation of the metallic vapor on its walls or upon the upper carbons. In order to avoid any difference in temperature at the poles of the vaporizing arc, or different electrical condition in the vapor rising from it, alternating currents of from 15 to 30 amperes were used. The vertical planes passing through each pair of carbons were at right angles to each other. The holes through which the carbons pass were packed with asbestos wicking.

The upper carbons, which were situated approximately 5 cm above the vaporizing arc, were supplied with direct current of from 20 to 60 amperes. The light from the upper arc passed out through an elliptical hole in the wall of the furnace. The image of the arc was cast on the slit of the spectrograph by a quartz lens, the image being linearly magnified approximately six times. After the furnace reached a high temperature its operation was very regular and free from trouble. However, when steam was admitted

to the furnace the carbons were used up rapidly and constant attention was necessary.

Most of the plates were taken on a concave grating spectrograph of 2 m radius, Rowland mounting, though some were obtained with a small spectrograph using either two prisms or a flat grating. On account of the astigmatism of the concave grating it was necessary to give two exposures to get photographs of spectra from two different portions of the arc. In order to eliminate any linearly progressive change in the arc during the time of exposure, one half the time of the first exposure was given, then all of the second, then the last half of the first, the camera shutter being turned between exposures; the arc itself and not the projection lens being shifted to bring the desired portion of the image on the slit at each exposure. That this process was successful was proved by photographing the same portion of the arc during the two exposures and by photographing the spectrum of an alternating-current arc near each electrode. The plates obtained were in both cases identical. With the magnification used, the breadth of the narrowest part of the image was considerably greate than the length of the slit of the spectrograph; hence the small unavoidable wandering of the arc had little or no influence. To decrease this wandering, cored carbons were generally used.

When the small prism spectrograph was used, it was turned over until the slit was horizontal, thus lying along the axis of the image of the arc. The size of the image was adjusted until its total length was smaller than the length of the slit, so that not only light from the arc stream but that from the tips of the electrodes entered the collimator. Thus the entire arc inclusive of the electrodes could be seen at one time.

In most of the work with the concave grating, spectra of the first order were used. In the blue and violet regions exposures of six minutes were sufficient, but in the red region, λ 6000 to λ 7000, exposures of fifteen to thirty minutes on Eastman Panchromatic films were necessary.

RESULTS

Relative intensities of lines at anode and cathode.—On the several hundred plates secured *there is not a single instance in which a*

metallic line is stronger at the anode than at the cathode, provided that the metal used is not an impurity in the carbons; and even when the metal is found as an impurity, *usually* its cathode lines are stronger than those at the anode.

If instead of surrounding the arc with a uniform atmosphere of metallic vapor, as is done here, the metal is introduced into the arc by loading the carbons, or by saturating them with solutions of the metallic salts, it is possible to have the lines strongest at either pole, depending upon which carbon has the greater supply of the element. In such cases it is probable that the vapor-density or the thickness of the vapor is greater near the more heavily loaded carbon. Several investigators have reported finding that certain metallic lines were stronger at the anode than at the cathode. So far as the writers are aware, in all such cases the metals were introduced by loading the carbons, by saturating them with solutions of the salts of the metals, or by depending upon the occurrence of the metal as impurities in the carbons. It may be that the difference between the results obtained by these investigators and that obtained in the present work is due to the fact that in the work of these investigators there may have been differences in the amount of the metals in the two electrodes.

As bearing upon this point, the following may be of interest. Using rods of wrought iron 12.5 mm in diameter for the upper electrodes and carbons loaded with $SrSO_4$ for the vaporizing arc, the plates show the *iron* lines of equal intensity at the anode and cathode, while the *strontium* lines were stronger at the cathode than at the anode.

The fact that the anode is hotter than the cathode has led several investigators to ascribe the stronger cathode lines to a process of distillation from the anode to the cathode with consequently greater vapor-density at the cathode. In order to investigate this matter, one of the upper carbons in the furnace was replaced by a water-cooled brass electrode. If there is a process of distillation in the arc whereby the vapors distil over from the hotter anode to the relatively cooler cathode, thus increasing the vapor-density in the vicinity of the latter, it ought to be possible to reverse the direction of the process by using an anode cooler

than the cathode. When the water-cooled electrode above mentioned is used as anode, the lines of the metals forming the vapor are as before, stronger at the cathode. In fact, the phenomenon is not interfered with in the least. Near the anode, however, the lines of the Cu and Zn of the electrode are stronger than they are at the cathode, though showing at both poles.

In order to determine whether or not the *gas of the arc stream* is cooler at the anode than at the cathode, two small thermopiles of the Rubens pattern were arranged so that they could be slid along a rail and either one could be placed at any desired point in the image of the arc. They were connected in opposition to each other and to a heavily damped galvanometer. The sensitiveness of the arrangement was tested by placing one thermopile near the image of the cathode while the other was moved to any desired portion of the arc. The difference of radiation from the different sheaths of the arc was easily shown.

The thermopiles showed plainly that with carbon electrodes the gas in the vicinity of the anode was hotter than that near the cathode. With the water-cooled electrode, however, the thermopile showed definitely and clearly that no matter whether this electrode formed the anode or cathode of the arc, the gas in its vicinity was cooler than the gas surrounding the carbon electrode.

Carbons bored out axially, the inner end of the boring not quite coming to the inner end of the carbon, and having the other end connected to the water supply, were also employed. The results were the same as with the brass tube.

The fact that the water-cooled electrode always lowers the temperature of the arc stream in its vicinity below that prevailing at the other electrode, taken in connection with the fact stated above that this electrode when used as an anode does not affect the greater intensity of the metallic lines near the cathode, seems to be evidence of considerable weight against the idea of distillation.

Blowing a gentle current of steam through a bored carbon in an attempt either to blow the metallic vapors away from the carbons or at least dilute them considerably resulted in a very badly burning arc, but no matter which electrode the steam-feeding carbon formed, the lines were stronger at the cathode.

Electrolysis.—The fact that when the vaporizing carbons are loaded with a mixture of elements standing as far apart in the electrochemical series as Li, K, Ba, Sr on the one hand, and Cu and Pb on the other, the lines of all of them are found to be stronger at the cathode, would seem to speak against the electrolytic theory. If there is electrolysis, it would seem that some of the elements mentioned should be found at one pole and some at the other. Experiment shows, however, that there is no separation of the elements in the arc.

If a third carbon loaded with these elements is pushed from the side into the arc, thus introducing the elements into the midst of the arc stream, there is no separation, the lines of all the metals being stronger near the cathode. This is true when the third electrode is connected to either of the electrodes through a moderate resistance, thereby changing the potential of the electrode with respect to the others.

If a fire-clay diaphragm with a small hole at its center is inserted in the furnace between the upper and lower carbons, thus making two chambers in the lower of which the vapor is formed, the vapor rises through the hole and crosses the arc stream as a jet of small diameter. It is found that still the metallic lines are strongest at the cathode, or rather between the cathode and the point where the jet impinges on the arc stream. As in the other arrangements just mentioned there is no separation of the elements. Owing to the somewhat indefinite outlines of the jet where it crosses or mingles with the arc stream, it could not be determined whether the lines were strongest at the cathode or at the boundary of the jet nearest the cathode.

Direct electrostatic action on the ionized gases surrounding the arc stream.—The term electrolysis was used above in the sense of implying an electrolytic transport of materials in the stream of the arc itself and by the action of the arc current. The term "direct electrostatic action on the ionized gases" is intended to convey the idea of an electrostatic action of the oppositely electrified electrodes upon the masses of vapor which surround the arc stream, but do not take part in conveying the arc current. It is possible at least that the ions coming from the lower arc, and

also to a certain extent from the upper one, would be attracted directly to the upper-arc electrodes, and, reaching them, would there give the spectral lines of the ionized elements.

To test this point, the lower arc was allowed to burn, furnishing a continuous supply of vapor, while the upper arc was extinguished and to its electrodes was connected a source of direct current of high potential. One electrode was thus at a high positive potential and the other negative, and both were immersed in the ionized vapors. After a run of from fifteen to twenty minutes, the source of high potential was disconnected, one of the deposition carbons was removed and a fresh carbon substituted for it. The arc current was now switched on, the new carbon being made the anode. A spectrum photograph was taken, the slit being located near the image of the deposition carbon. This carbon was now removed and the other deposition carbon substituted for it, and a second spectrum was photographed on the same plate. Using potential gradients of 200 to 5000 volts per cm of distance between the ends of the deposition carbons during the deposition, the spectrum plates were identical in lines and in the intensities of the same.

As a variation, the vapors from the arc are allowed to pass through an iron tube about 3.5 cm in diameter and about 40 cm long, in the axis of which was a brass rod 0.5 cm in diameter insulated from the iron tube by a porcelain bushing tube. A difference of potential of from 1000 to 2000 volts between the rod and the tube was applied, and after a suitable interval for deposition the rod and tube were removed, separated, and each wiped off with a wad of absorbent cotton. The wads of cotton were placed in separate beakers and each treated with hot dilute HCl in order to dissolve whatever had been deposited on the tube or rod and removed by the wads of cotton. New carbons were boiled in the solutions. These carbons formed in succession the cathodes of an arc whose cathode spectrum was photographed, the anode being a new carbon. The result of these photographs was that the spectrum lines of the metal used in the vaporizing arc were absolutely identical in the photographs. The spectral reaction was delicate enough to show the iron lines in the material deposited

on the iron tube, and the copper lines in the material deposited on the brass rod, these being due to the exceedingly small particles of the oxides wiped off by the wads of cotton. Using a mixture of elements in the vaporizing carbon, there was in no case any separation of these elements in this process, the materials deposited of the high potential anode and cathode being identical.

It is the opinion of the writers that the facts above given constitute a considerable body of evidence against any of the thermal processes and also against electrolysis and direct electrostatic action, and consequently of the various explanations offered there remains only the electronic one offered by Humphreys.

It can be said at once that although the work described above cannot be considered a proof of the electronic theory, yet there is nothing in its results in conflict with this theory. The following appears to be stronger and more direct evidence than that just given. An inclined carbon tube was arranged so that small pellets of the metallic salts from 1 to 2 mm in diameter could be fed into the upper end, gain considerable speed in the descent, and finally plunge through the arc stream at high speed. The time that these pellets were in the arc stream was too short for any perceptible diffusion, convection, or electrolysis, and further, since their diameters were so small there could not have been much difference of potential or difference of temperature between the side turned toward the cathode and that turned toward the anode. A plane-grating spectroscope with its slit horizontal was used to study visually the spectrum given by these pellets. Under the conditions mentioned, the lines of the metals were distinctly *fish-shaped*, the *head or thickest part of the fish being turned toward the cathode.* This suggests that the electrons, flying from the cathode, strike the vapor surrounding the pellet on the side turned toward the cathode, produce the light, and then, most of them being stopped by collision either with the vapor or the solid body of the pellet itself, there are only comparatively few left to produce light on the anode side. It seems that it would be difficult to explain this result on any other theory than that of ionization by impact.

Explanations based on convection, diffusion, distillation, or electrolysis are alike in that they postulate an accumulation of

metallic vapors in the vicinity of the cathode. In accordance with any of these explanations, if the polarity of a direct-current arc is reversed, the metallic lines from the new cathode cannot be more intense than those from the new anode until sufficient time has elapsed for the accumulation left around the new anode to diminish and for a new accumulation of vapor to build itself up around the new cathode. It is evident that such a process requires a finite time, since it requires a transfer of ordinary matter. On the other hand, on account of the high velocity of electrons electronic processes are almost instantaneous, and reversing the arc merely means reversing the direction of electron flow. Very little time is required for this, evidently far less than for the accumulation of metallic vapor around one of the poles of the arc. Hence the time required after the reversal of the arc for the lines to become more intense at the new cathode may be used as a means of deciding between the electronic and rival theories above mentioned.

In order to apply this method of discrimination, the following scheme was used. Sixty-cycle alternating current was used to operate the upper arc in the furnace, the "vaporizing carbons" as usual furnishing the metallic vapor. Each upper carbon was consequently the anode for 1/120 of a second, followed by an equal interval of time during which it was the cathode, and so on. Between the projection lens and the slit of the spectrograph was placed a small synchronous motor of 1800 r.p.m. on whose shaft was mounted a sectored metallic disk. The disk had two open and two opaque sectors. A contact device on the shaft of the motor insured that the beam of light traversed the center of one of the open sectors at the instant that one carbon was at its highest position potential and the other at its maximum negative potential. When the current through the arc reversed, an opaque sector had taken the place of the open one, thus shutting off the light. Since the open sectors were appreciably smaller than the opaque ones, there could be no overlapping.

Under these conditions the image of the alternating-current arc and the sheaths were in all respects precisely the same as those of a direct-current arc.

Spectrum photographs showed the familiar phenomenon of the increased intensity of the metallic lines at the cathode *precisely as if the arc had been produced by a direct current.* With the vaporizing carbons loaded with a mixture of the salts of Li, K, Sr, Ba, and Pb, the lines of all of these elements were stronger at the cathode. This shows without doubt that 1/120 of a second is ample time for the phenomenon to develop, and hence seems to exclude any possibility of there having been any bodily transference of metallic vapors from one pole to the other. It would probably be possible, by the use of narrower open sectors and by revolving the field frame of the motor to such a position as to allow the light to pass at an instant very nearly that at which the current passes through zero, to obtain a more definite value for the time limit required for the establishment of the phenomenon, but it would seem that a maximum limit of 1/120 of a second is sufficient to show that there could be little or no actual transference of metallic vapors from one side of the arc to the other.

The fact that elements of such widely different atomic weights, ranging from 7 to 208, all gave the same result under these conditions of rapid alternations of polarity, appears to furnish additional evidence against any explanation or theory involving the idea of transference of material across the arc, and to strengthen the supposition that the light emitted by the arc stream is due to vibration caused by the collision of electrons emitted by the cathode with the particles of the vapors surrounding the arc.

Relative intensities.—The different lines of a given element show considerable differences in their relative intensities at the two poles. Thus some Sr lines may have an intensity of 10 at the cathode and 8 at the anode, while others may have intensities of 20 and 1, and in some cases no trace whatever of the line could be seen at the anode. An attempt has been made to see whether there is any connection between this change of intensity in the line at the two poles and the Kayser and Runge series to which the line might belong. It has not, however, been possible to reach any definite decision on this point.

A similar question is that of correlating the behavior of the lines with the atomic weight of the element. While the number

of elements used was too small to allow any definite statement to be safely made, yet, so far as the results go, they indicate that the cathode strengthening is greater the lower the atomic weight of the element.

Conclusion.—All attempts to separate vapors of the arc by methods involving difference of temperature and by methods based on electrolysis and electrostatic action gave negative results. Results obtained with the alternating-current arc seem to show that the light from the arc stream is due to electronic collision, and they indicated that the phenomenon is completely established in 1/120 of a second or less.

Alabama Polytechnic Institute
Auburn, Alabama
January 12, 1921

Minor Contributions and Notes

IDENTIFICATION OF AIR LINES IN SPARK SPECTRA FROM λ 5927 TO λ 8683[1]

ABSTRACT

Spectrum of condensed spark in air and in oxygen, λ 5927 to λ 8683.—By operating the spark alternately in oxygen and in air, the lines due to oxygen have been identified. The remaining air lines, except a few *argon* lines, are ascribed to *nitrogen*. The work confirms numerous identifications made by other observers using vacuum tubes, and adds some identifications not previously available.

Experiments in continuation of those previously performed by one of the authors[2] have been carried out to determine the chemical identifications of the air lines from λ 5927 to λ 8683 in spark spectra.

The spark chamber, consisting of a glass bulb, was connected with an air pump and an oxygen apparatus in such a way that it could be filled either with air or with oxygen. The oxygen was generated by heating potassium permanganate and was purified by passage through caustic potash, calcium chloride, and phosphorus pentoxide. The photographic exposures were so arranged that the spectrum of the spark in air appeared on the negative close beside that of the same spark in oxygen, thus facilitating a comparison of the relative intensities of the lines. The pressure in the spark chamber was equal to one atmosphere in all cases. Several sets of comparisons were made with copper electrodes, and one with carbon (Acheson graphite) electrodes. All gave accordant indications of the identification of the air lines.

The second column in Table I gives our observations of the changes in relative intensities of the lines produced by the spark in oxygen as compared with the spark in air, *w* and *s* indicating weakened and strengthened, respectively. Of lines for which other identifications are available, those due to argon and nitrogen are weakened, while those due to oxygen are strengthened. Among the remaining lines, those weakened are believed to be due to nitrogen, and those strengthened to oxygen.

[1] *Contributions from the Mount Wilson Observatory*, No. 207.

[2] *Ibid.*, No. 183; *Astrophysical Journal*, **51**, 211, 1920.

One line, λ 6370.92, should be removed from the previous list,[1] as it is probably due to silicon, while a nitrogen line at about 6341.5 A should be added, as it is observable in air. It undoubtedly corresponds to a line observed at 6340.84 A by Porlezza in a nitrogen tube.

TABLE I

RELATIVE INTENSITIES OF SPARK LINES IN AIR AND IN OXYGEN

λ in Air, I A	Oxygen	Vacuum Tubes	λ in Air, I A	Oxygen	Vacuum Tubes
5927.83.....	*w*	N*	7424.04.....	*w*	....
5931.78.....	*w*	N*	7442.70.....	*w*	....
5941.54.....	*w*	N*	7468.72.....	*w*	....
5952.33.....	*w*	N*	....	*s*	O 7476.58‡
6158.	*s*	O†	....	*s*	O 7479.23‡
6171.0	*w*	....	....	*s*	O 7481.27‡
6284.22.....	*w*	N*	7635.70.....	*w*	A
(6341.5).....	*w*	N 6340.84*	7772.07.....	*s*	O
6358.13.....	*w*	N*	7774.33.....	*s*	O
(6370.92)....	....	See note	7775.60.....	*s*	O
6379.52.....	*w*	N*	7947.83.....	*s*	O 7947.58‡
6456.	*s*	O†	7951.10.....	*s*	O 7950.84‡
6482.054....	*w*	N*	7952.3	*s*	O 7952.22‡
6610.39.....	*w*	N*	8185.26.....	*w*	....
6640.7	*s*	....	8188.42.....	*w*	....
6654.78.....	*s*	....	8200.72.....	*w*	....
6721.25.....	*s*	....	8211.12.....	*w*?	....
6887.61.....	*w*	....	8216.72.....	*w*	....
6950.	*w*?	....	....	*s*	O 8221.84‡
6965.95.....	*w*	A	8223.48.....	*w*	....
(7007)	*s*	O 7002†	8230.20.....	*s*	O 8230.05‡
7067.6	*w*	A	....	*s*	O 8233.05‡
7157.36.....	*s*	O?†	8242.80.....	*w*	....
....	*s*	O 7254†	8446.84.....	*s*	O
7384.53.....	*w*	A	8630.02.....	*w*?	....
			8680.63.....	*w*	....
			8683.70.....	*w*?	....

* Porlezza, *Atti R. Accademia dei Lincei*, Serie 5, **20**, 584, 1911.

† Runge and Paschen, *Annalen der Physik*, **61**, 641, 1897; **27**, 562, 1908.

‡ Kiess, *Popular Astronomy*, **29**, 18, 1921.

NOTES

λ 6158 Very broad, probably double or triple; includes λ 6160.72.

6171.0 Broad, probably double or triple.

6370.92 Not an air line; probably Si.

7007 Very, very hazy.

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[1] *Loc. cit.*

AVOIDANCE OF ATMOSPHERIC DISPERSION IN MEASURES WITH THE STELLAR INTERFEROMETER

ABSTRACT

Stellar interferometer.—A direct-vision prism has been used with the stellar interferometer to avoid the disturbing effects of atmospheric dispersion.

Since Professor Michelson's tests here and at Mount Wilson have shown that the interference method for measuring close double stars is little affected by atmospheric conditions, we have applied this method to some of the known bright pairs with the 40-inch refractor. An experimental apparatus was built in 1920 by Mr. O. J. Lee and myself, with the use of wood and paper, because of lack of help in our shop at that time. It was placed in front of the 40-inch objective and consisted of two rectangular openings in cardboard that could be changed in position angle and varied in distance by means of cords held by the observer. This crude instrument showed the expected phenomena very plainly, but was only good enough to make a better one desirable. From a sketch by myself a small apparatus was made of iron and brass in which the movable slits were in the cone of light only one meter inside the focus. This instrument was mainly made by Dr. George S. Isham, of Chicago, who voluntarily proposed to undertake the construction in his private shop. His hearty co-operation is very cordially acknowledged here. Adapted to the 40-inch refractor by our instrument-maker, with the necessary adjustments for centering the apparatus in the cone of light, the instrument proved satisfactory. Between March 21 and June 25, 1921, Capella has been followed in position angle over nearly a complete revolution by observations during the day, some of them when Capella was culminating at noon.

From the very beginning, however, a difficulty was encountered in atmospheric dispersion. J. A. Anderson[1] has clearly explained the effect of this disturbing factor. When the interferometer is set on a star at low altitude the fringes appear fan-shaped if the slits are vertical, while in a direction normal to this the fringes would be completely blurred. Without going that far, it is evident that even at higher altitudes atmospheric dispersion will affect the change in

[1] *Astrophysical Journal*, 51, 268, 1920.

visibility of the fringes for varying position angles. The effect would be an apparent shift of the position angle toward the vertical. These deviations were quite evident on Capella. This star is too close to be measured in distance with the 40-inch refractor combined with the interferometer; yet the change of visibility of the fringes when the instrument is turned in position angle is such that, although the distance could not be measured, the position angle can be established fairly well. But the measures on a given day showed marked changes with the hour angle.

Anderson compensates for the atmospheric dispersion by putting plates of plane-parallel glass, one of them being slightly tilted, in front of each slit; but this appeared difficult, since the adjustment had to be modified for each change in the position angle. The alternative of using a prism of variable angle, kept in the required position by gravity, will probably be found too difficult in actual application.

Another method was devised by myself, which is simple and practical. It consists in mounting a direct-vision prism on the eyepiece, the edge of the prism being parallel to the line joining the centers of the slits, so that the dispersion is parallel to the slits. Since the eyepiece turns with the slits, the prism becomes part of the interferometer and does not require repeated adjustment. Direct-vision prisms of the Zöllner type, consisting of two crown prisms with the flint prism in between, were used for that purpose, the cylindrical lens being discarded. Dispersions of 3°, 4°, and 5° respectively, between the Fraunhofer lines B and G (λ 6868 and λ 4308) proved to be amply sufficient.

Atmospheric dispersion is represented by $0\rlap{.}''83 \tan z$ for zenith distances smaller than 70° if the limits of the spectrum are taken at λ 6500 and λ 4500.[1] The corresponding value between B and G is $1\rlap{.}''06 \tan z$. Since the observations are made with a power of about 2000, the atmospheric dispersion would appear to the eye under an angle of $2000 \times 1.06 \tan z$ or roughly $0\rlap{.}^\circ 7 \tan z$, as against a dispersion of 3° to 5° with the prism. The combination of the two dispersions will be a spectrum at a slight angle with the length of the slits. By adjusting the prism so that the resulting spectrum is

[1] Anderson, *loc. cit.*

parallel to the slits, the vertical dispersion is compensated and the fringes will be seen again over the whole length of the successive colors of the narrow spectrum, especially well in the red part for the prism used. Laboratory tests in which the light of a slit-source was dispersed through a 60° prism showed that even for such a high dispersion the lost fringes could be brought back and made so plain by the direct-vision prism oriented in the proper way that the width of the slit could be measured as if there was no dispersion, and since the settings could be made in the various colors, there is no uncertainty as to the wave-length to be used in the computations.

This method was used on Capella for the first time on June 16 and was found to work very well even when the star is as close to the sun as at present. On fainter stars the loss of light would reduce the number of objects that can be reached. Observations on stars as faint as $7^{M}.0$ were found possible under fair conditions, without the direct-vision prism. The dispersion introduced by this auxiliary apparatus would put the limit somewhere near $5^{M}.0$ with the actual arrangement of the instrument.

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June 26, 1921

ERRATA

Vol. **53**, January, 1921, "The Parallaxes of 1646 Stars Derived by the Spectroscopic Method," by W. S. Adams, A. H. Joy, G. Strömberg, and Cora G. Burwell:

Page 23, equation (19), *for* $\bar{A}^2$ *read* $\overline{A}^2$.

Page 26, second line, *for* below *read* above.

Page 39, Boss 451: *for* 47 Cassiop. *read* 57 Ceti.

Page 44, the trigonometric π and authorities given for Boss 1074 refer to C594.

Page 60, first line, the Boss No. for Bu. 5695C should be deleted.

Page 75, Boss 4470: under "Authorities," *for* M *read* S.

Page 84, X Cygni: in the sixth column, insert ‖ after 0.018.

Page 87, C2797: in the sixth column, *for* 0."056 *read* 0."56.